LOFO 2020

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1A. Prove that $(\neg P \lor Q) \Leftrightarrow (P \Rightarrow Q)$ is a tautology.

1B. Prove that $\neg(P \lor Q) \Leftrightarrow (\neg P \land \neg Q)$ is a tautology.

2A. Is the following Hilbert system complete with regards to $\mathcal{F}_{\{\bot,\neg,\lor,\Rightarrow\}}$?

$$\frac{A \Rightarrow B \qquad A}{B}\,[\textit{Modus Ponens}]$$

$$\frac{}{\bot \Rightarrow A}\,[\bot] \qquad \frac{}{A \Rightarrow B \Rightarrow A}\,[\Rightarrow_1] \qquad \frac{}{(A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,[\Rightarrow_2]$$

$$\frac{}{(A \Rightarrow \bot) \Rightarrow \neg A}\,[\neg_1] \qquad \frac{}{A \Rightarrow \neg A \Rightarrow \bot}\,[\neg_2]$$

2B. Is the following Hilbert system complete with regards to $\mathcal{F}_{\{\bot,\land,\lor,\Rightarrow\}}$?

$$\frac{A \Rightarrow B \qquad A}{B}\,[\textit{Modus Ponens}]$$

$$\frac{}{A \Rightarrow A \lor B}\,[\lor_1] \qquad \frac{}{B \Rightarrow A \lor B}\,[\lor_2] \qquad \frac{}{A \lor B \Rightarrow (A \Rightarrow C) \Rightarrow (B \Rightarrow C) \Rightarrow C}\,[\lor_3]$$

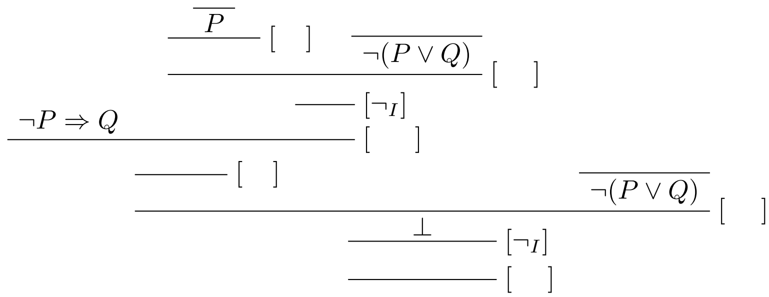
$$\frac{}{A \land B \Rightarrow A}\,[\land_1] \qquad \frac{}{A \land B \Rightarrow B}\,[\land_2] \qquad \frac{}{A \Rightarrow B \Rightarrow A \land B}\,[\land_3]$$

$$\frac{}{A \Rightarrow B \Rightarrow A}\,[\Rightarrow_1] \qquad \frac{}{(A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,[\Rightarrow_2]$$

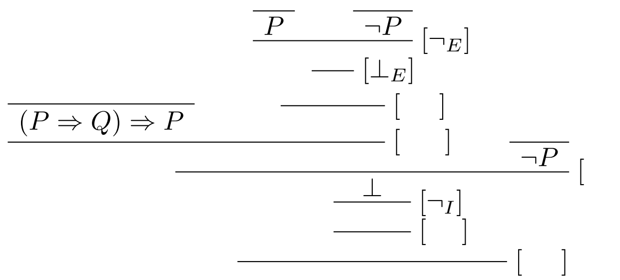
3A. Prove that $\{Q \land P, R\} \vdash_{\mathcal{N}} P \land (R \land Q)$. This can be done using a proof of depth 4.

3B. Prove that $\{P \lor Q\} \vdash_{\mathcal{N}} P \lor (Q \lor R)$. This can be done using a proof of depth 4.

4A. Prove that $\{\neg P \Rightarrow Q\} \vdash_{\mathcal{N}} P \lor Q$ by filling the blanks of the following tree:



4B. Prove that $\vdash_{\mathcal{N}} ((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$ by filling the blanks of the following tree:



5A. Define a term Square $\in \Lambda$ such that for any natural integer n:

$$\text{Square}\ \underline{n} \rightarrow_{\beta}^{*} \underline{n^2}$$

Then guess its type.

5B. Define a term Double $\in \Lambda$ such that for any natural integer n:

$$\text{Double}\ \underline{n} \rightarrow_{\beta}^{*} \underline{2 \times n}$$

Then guess its type.

6. Prove that $\Theta = (\lambda xy \cdot y(xxy))(\lambda uv \cdot v(uuv))$ is a fixed-point combinator.

7. Prove that $\vdash KI : \tau \rightarrow \sigma \rightarrow \sigma$.

8A. Prove that $\vdash_{\mathcal{NI}} (P \wedge Q) \Rightarrow (Q \wedge P)$.
Then find a term in Λ_{ext} of type $\sigma \times \tau \rightarrow \tau \times \sigma$.

8B. Prove that $\vdash_{\mathcal{NI}} (P \vee Q) \Rightarrow (Q \vee P)$.
Then find a term in Λ_{ext} of type $\sigma \cup \tau \rightarrow \tau \cup \sigma$.