

**Physics Midterm n°1***Calculators and extra-documents are not allowed.**Answer only on the exam sheets.*

Remember that, except if explicitly written in the questions, the notation  $E_A(M)$  corresponds to the norm of the field  $\vec{E}_A(M)$ . But the angles are oriented.

We will use for convenience the usual constant  $k = \frac{1}{4\pi\epsilon_0}$ .

**MCO** (4 points-no negative points) Circle the right answer.

1- The electric field, which is generated by a pointlike charge  $q$  located at point O, at some point M reads:

a)  $\vec{E}(M) = k \frac{q}{OM^2} \vec{OM}$       b)  $\vec{E}(M) = k \frac{q}{OM^3} \vec{OM}$       c)  $\vec{E}(M) = k \frac{q}{OM} \vec{OM}$

2- We are looking at the electrostatic force  $\vec{F}_{1 \rightarrow 2}$  that a charge  $q_1$  located at A creates on a charge  $q_2$  located at B. The norm of this force is given by:

a)  $F_{1 \rightarrow 2} = k \frac{q_1 q_2}{AB}$       b)  $F_{1 \rightarrow 2} = k \frac{|q_1| |q_2|}{AB}$       c)  $F_{1 \rightarrow 2} = k \frac{|q_1| |q_2|}{AB^2}$

3- The electrostatic force is a force which is:

a) Always attractive      b) Always repulsive      c) Always conservative

4- Which property does the electrostatic field  $\vec{E}$  derived from the potential  $V$  satisfy?

a)  $\vec{E} = -\overrightarrow{\text{grad}}(V)$       b)  $\vec{E} = \overrightarrow{\text{grad}}(V)$       c)  $V = \overrightarrow{\text{grad}}(\vec{E})$

5- Consider a surfacic distribution of charges  $\sigma$ . An infinitesimal surface element  $dS$  located at point P creates at some point M, where there is a charge  $q$ , an elementary force  $\vec{dF}$  which reads:

a)  $\vec{dF} = kq \frac{\sigma dS}{PM} \vec{PM}$       b)  $\vec{dF} = kq \frac{\sigma dS}{PM^3} \vec{PM}$       c)  $\vec{dF} = kq \frac{\sigma dS}{PM^2} \vec{PM}$

6- Consider a uniform surfacic distribution of positive charges  $\sigma$  on a cylinder of axis (Oz), of radius  $R$  and of length  $h$ . Which infinitesimal surface element  $dS$  is not relevant for this geometry?

a)  $dS = r dr d\theta$       b)  $dS = dx dy$       c)  $dS = r d\theta dz$

7- We are looking at the limit case of an infinite cylinder of axis (Oz) (with a unitary vector  $\vec{u}_z$ ), which is uniformly positively charged at its surface. We want to get the electric field  $\vec{E}(M)$ , where  $M$  is located on the (Oz)-axis. What can be claimed?

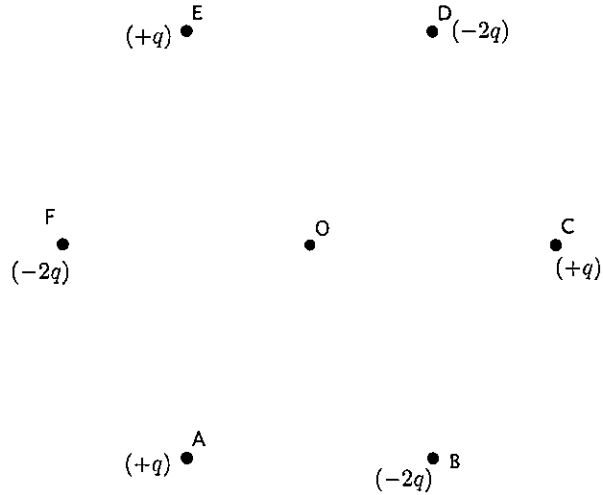
a)  $\vec{E}(M) = \vec{0}$       b)  $\vec{E}(M) \cdot \vec{u}_z > 0$       c)  $\vec{E}(M)$  is divergent.

8- Consider again the finite cylinder of the question 6. At some point  $M$  outside the cylinder, the cylindrical components ( $E_\rho, E_\theta, E_z$ ) of the electrostatic field satisfy:

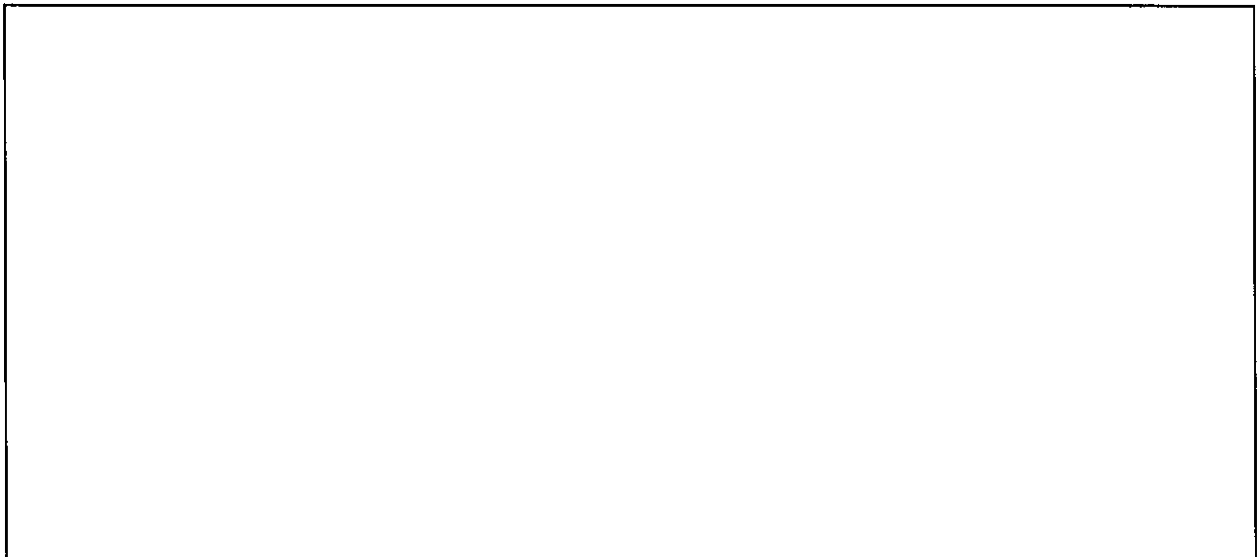
a)  $E_\rho = 0$       b)  $E_\theta = 0$       c)  $E_z = 0$

**Exercise 1**

Consider the following charge distribution ( $q > 0$ ), which forms a regular hexagon with edge length  $a$  and center  $O$ .

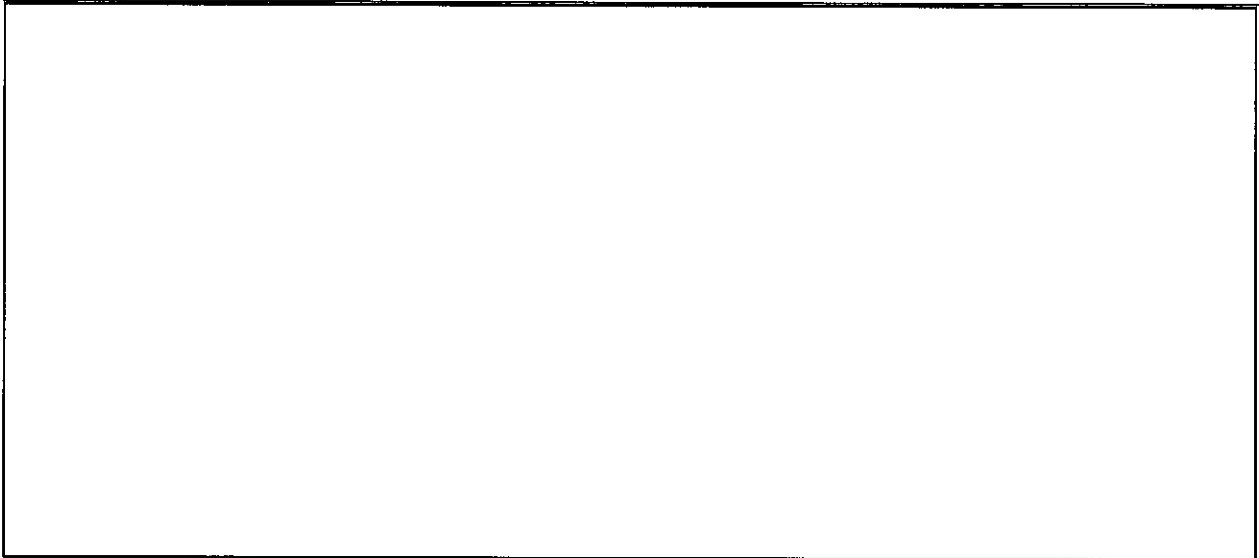


1- a) Express the electrostatic fields  $\vec{E}_A(O)$ ,  $\vec{E}_C(O)$ ,  $\vec{E}_E(O)$  created at  $O$  by the charges which are respectively at points  $A, C$  and  $E$ . Sketch them on the drawing above.

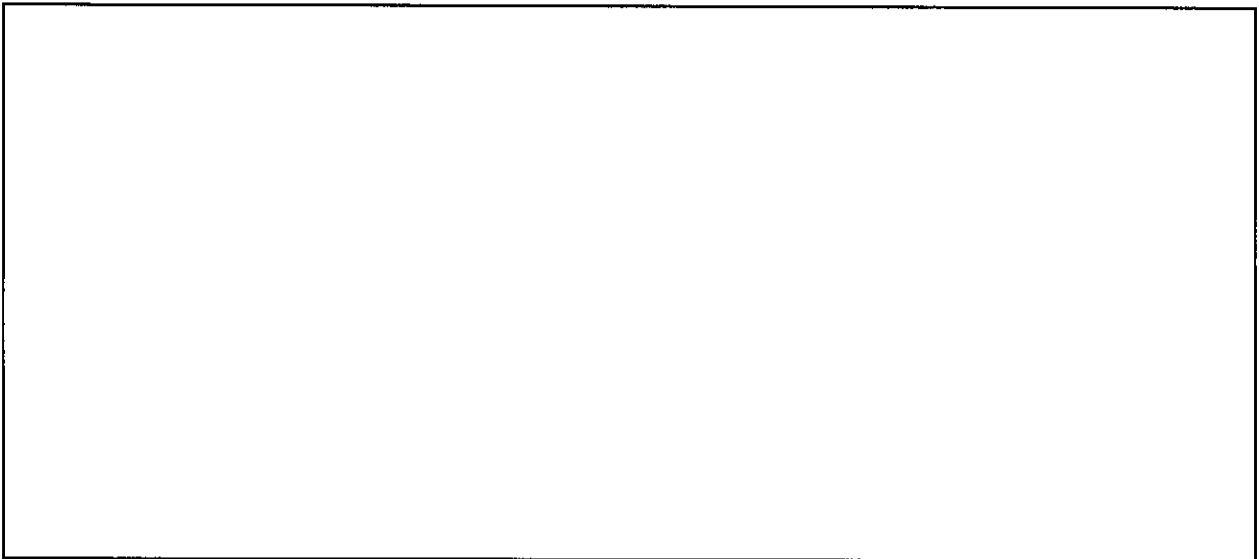


b) Compute the norm of the total electrostatic field created by these three charges at point  $O$ .





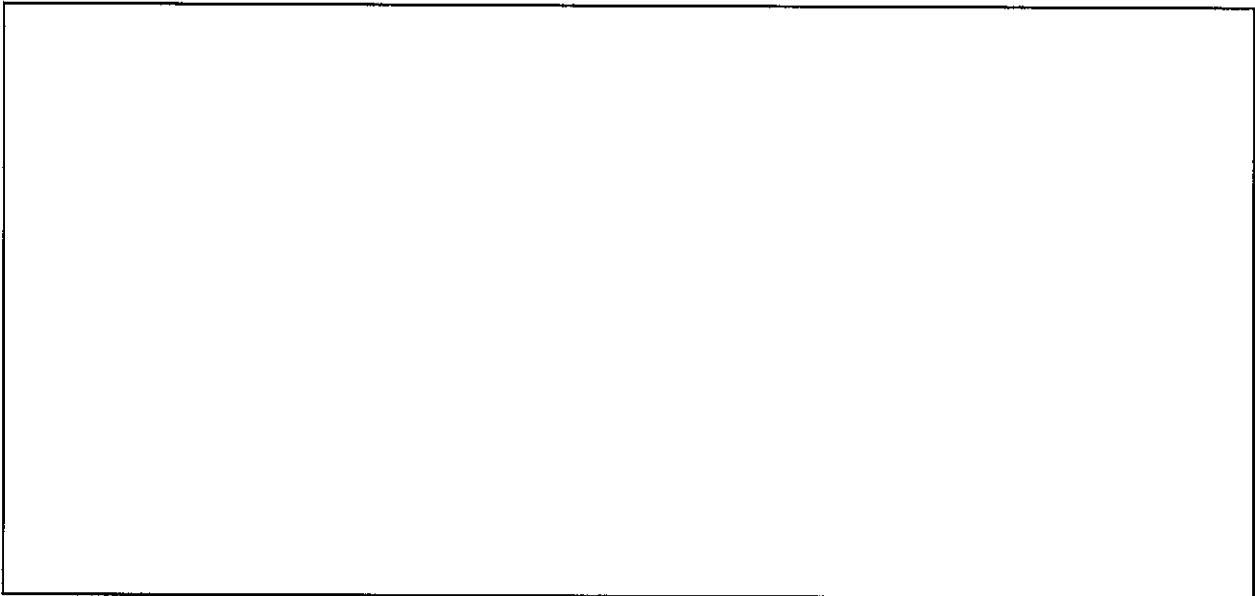
2- a) Let us put a charge  $Q < 0$  at point O. First sketch the force generated by the charges which are at A, C and E on charge  $Q$ . Then express the norm of this force.



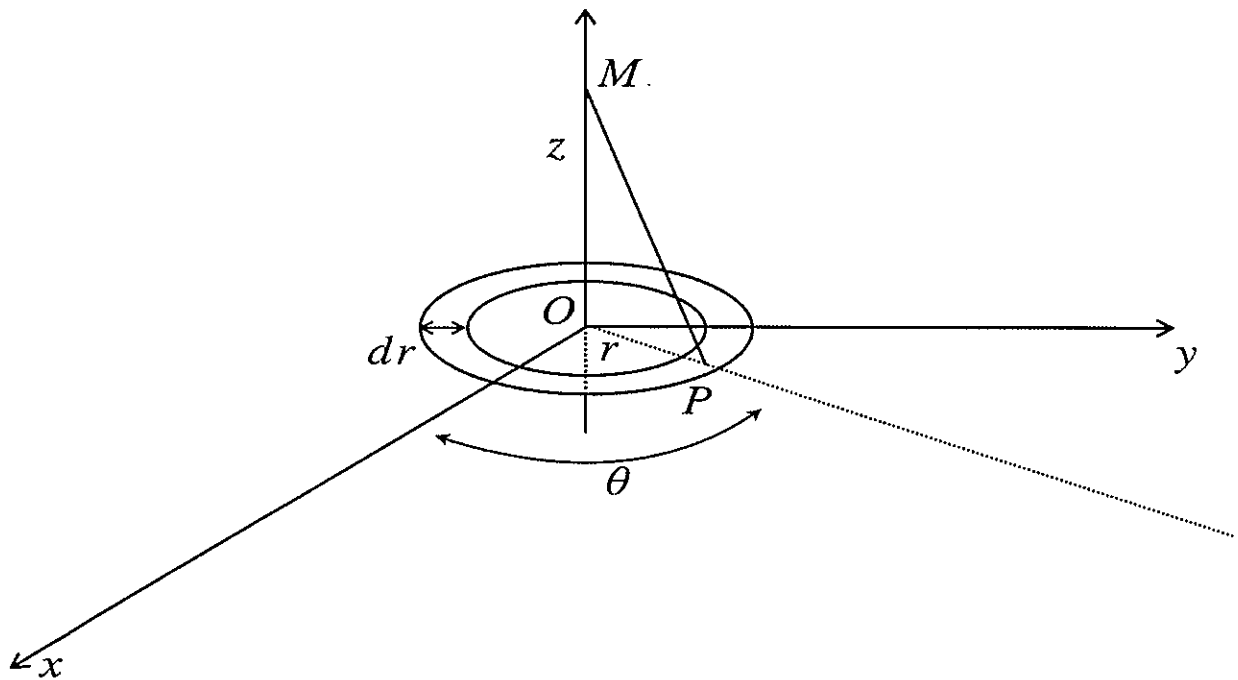
b) Express the electrostatic potential  $V(O)$  created at O by the charges located at B, D and F.



c) By taking into account all the charges located at the tips of the hexagon, give the expression of the electrostatic potential energy  $\mathcal{E}$  of the charge  $Q$  located at O.



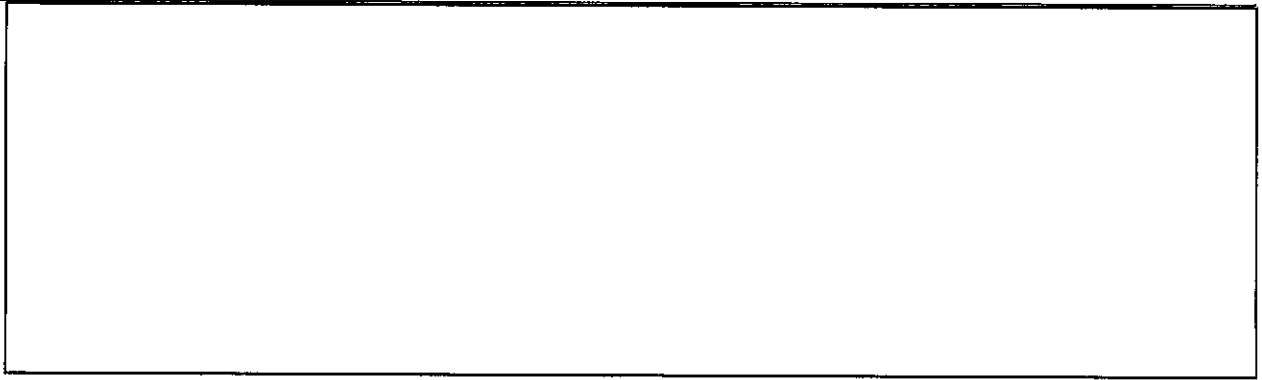
### Exercise 2



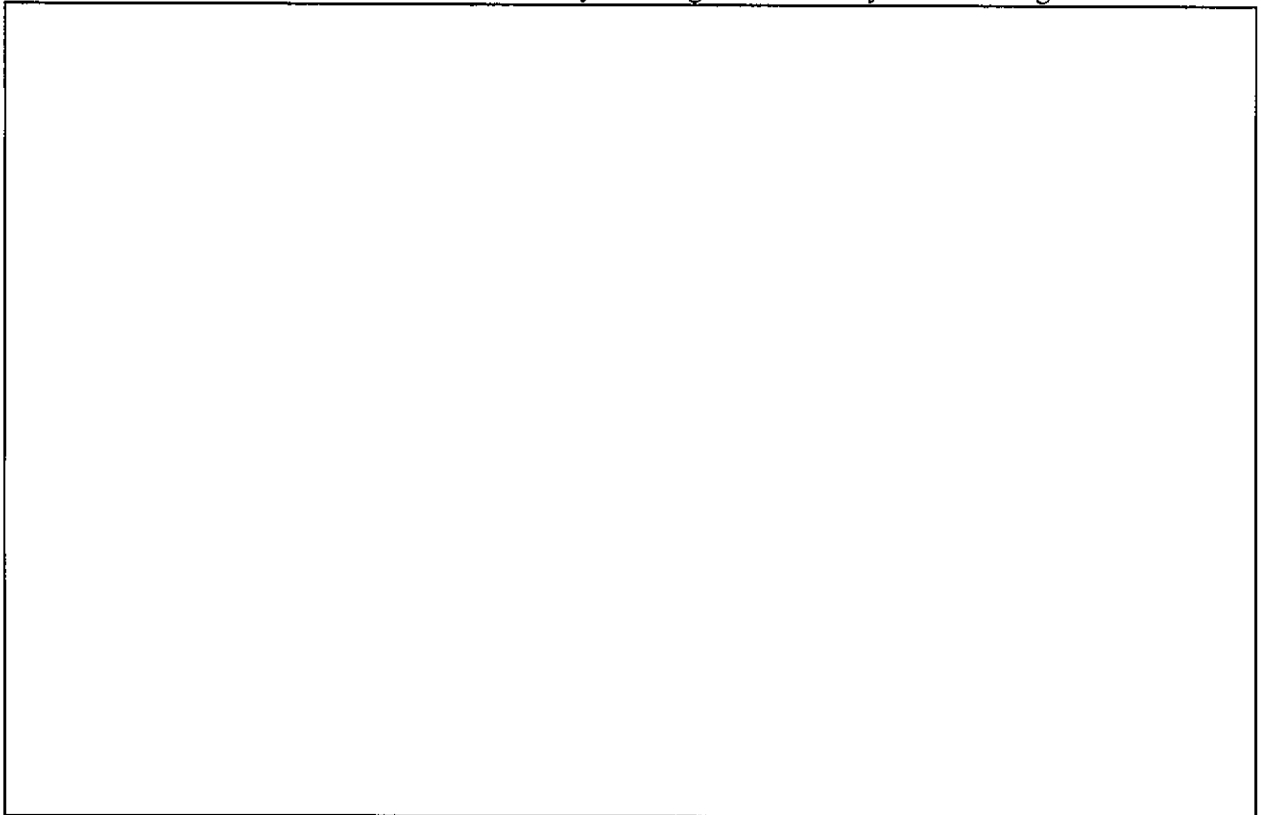
Consider a surfacic distribution of charges  $\sigma$ , which is uniformly distributed on a ring of radius  $r$ , of width  $dr$  and center  $O$ . The point  $M$  is on the  $(Oz)$ -axis.

1- Give the expression of the elementary electrostatic field  $d\vec{E}_P(M)$  created at  $M$  by an elementary surfacic charge  $dQ$  centered at P.



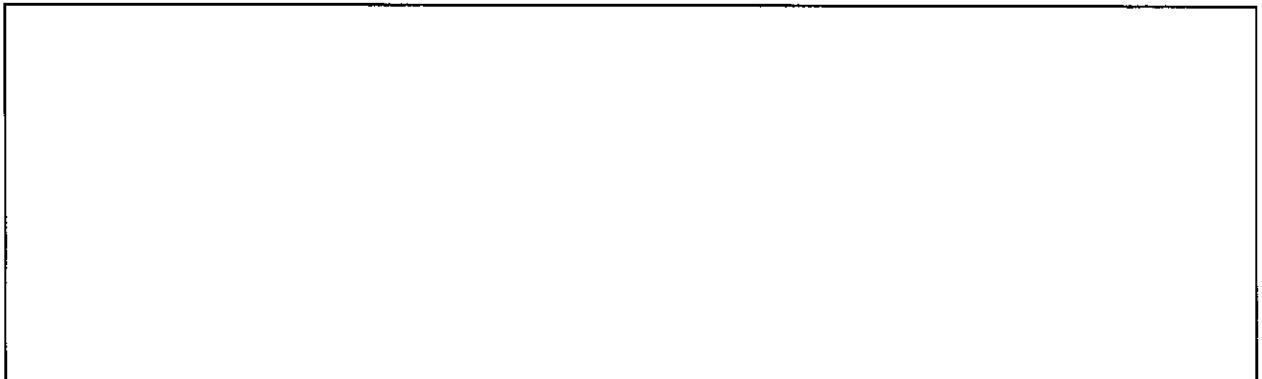


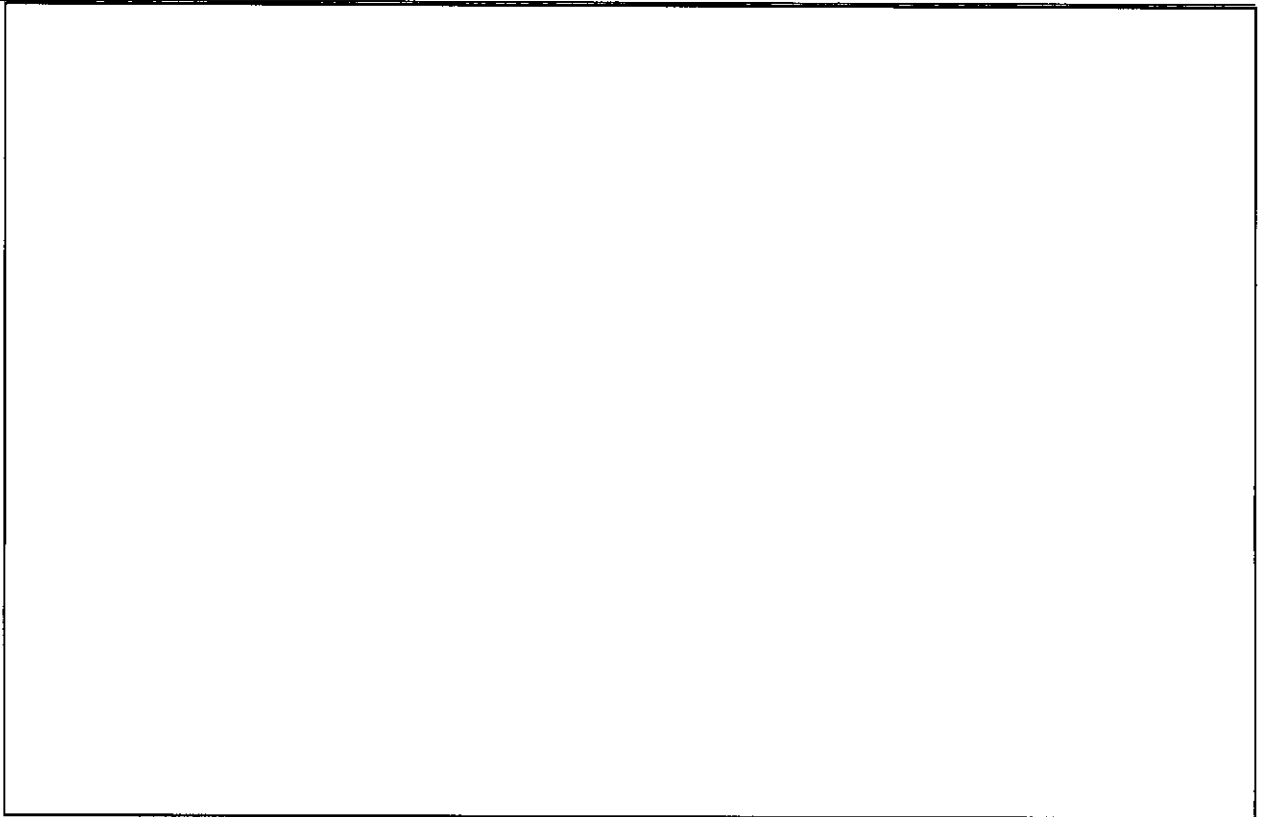
2- Deduce the total electrostatic field created by this ring at  $M$ . Detail your reasoning.



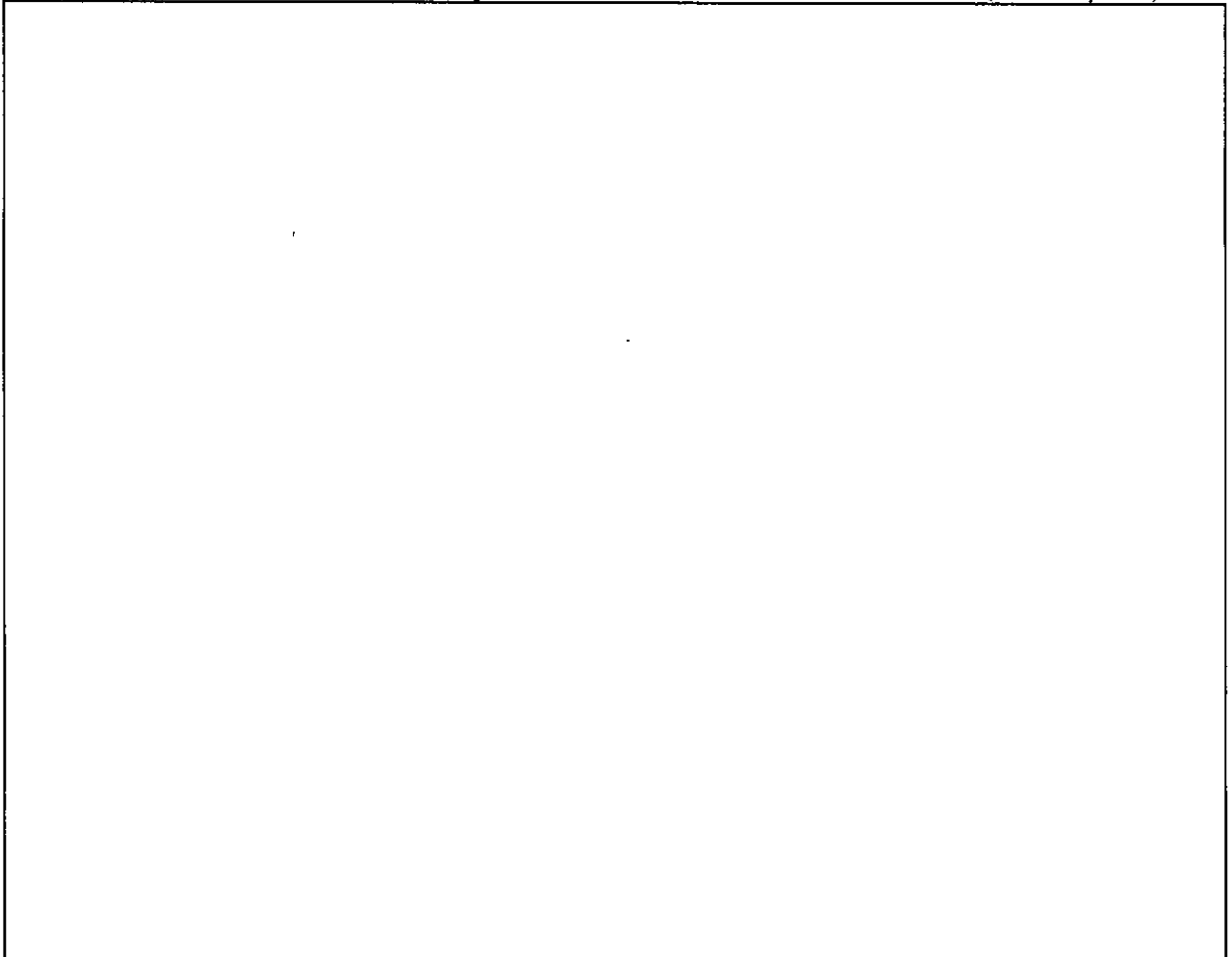
3- We aim to determine the electrostatic field  $\vec{E}(M)$  created at  $M$  by a disc of radius  $R$ , of center  $O$  and axis  $(Oz)$ .

By using the question 2, first recover the expression  $\vec{E}(M) = 2\pi k\sigma z \left( \frac{1}{|z|} - \frac{1}{\sqrt{R^2+z^2}} \right) \cdot \vec{u}_z$ , and then its norm.





4- By using the symmetries of the charge distribution, comment the limit  $R \rightarrow \infty$  (infinite plane).

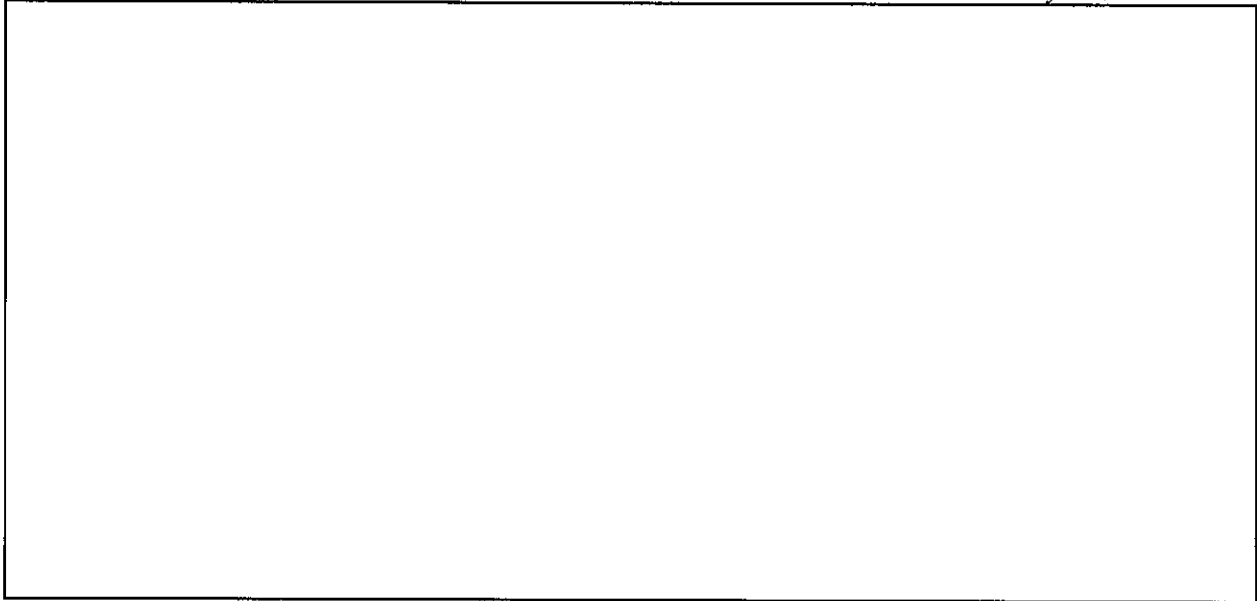


**Exercise 3**

The electrostatic potential  $V(x, y, z)$  is given in Cartesian coordinates by the following expression

$$V(x, y, z) = k \frac{q}{\sqrt{x^2 + y^2 + z^2}}.$$

1- Express the electric field  $\vec{E}(x, y, z)$  deriving from this potential in the basis  $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$ .



2- Recover, using the question 1, the expression of the unitary radial vector  $\vec{u}_r$  of the spherical basis. You can use the following expression of the gradient given in spherical coordinates, where  $f(r)$  is a function of the coordinate  $r$ :  $\overrightarrow{\text{grad}} f = \frac{\partial f}{\partial r} \vec{u}_r$ .

