

Name :

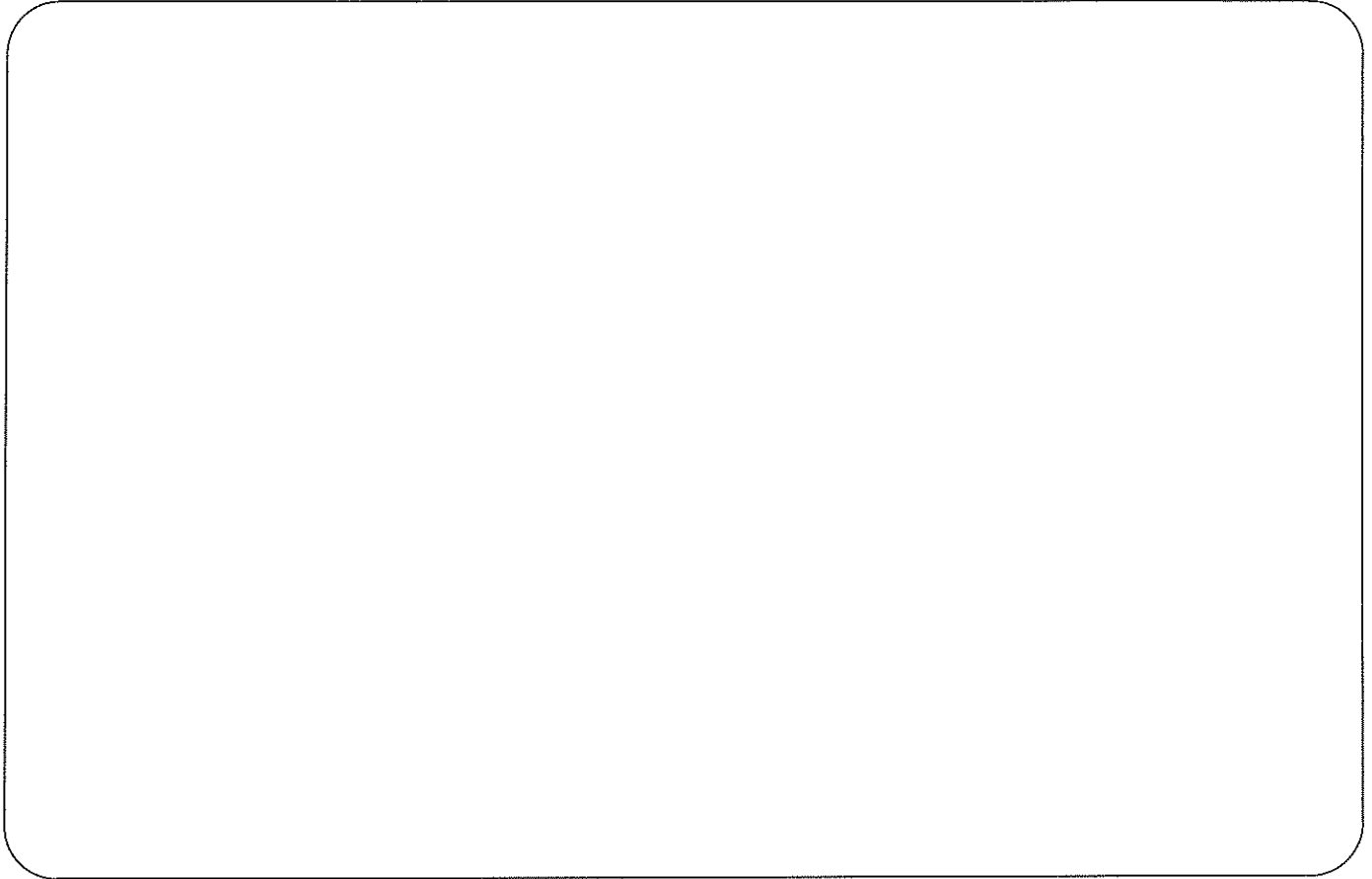
First Name :

Class :

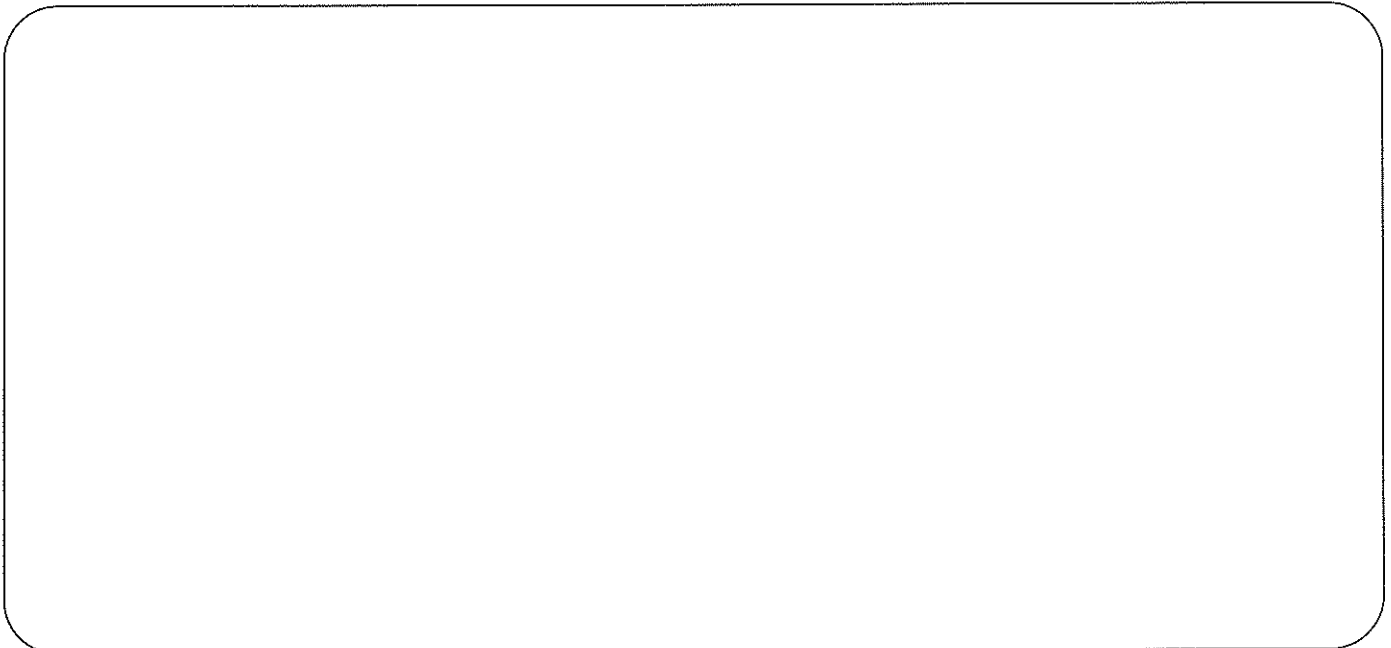
# Midterm exam n°1

## Exercise 1 (3 points)

1. Determine the Taylor expansion around 0 at order 3 of  $\ln(2 + \sin(x))$ .



2. Determine  $\lim_{x \rightarrow +\infty} \left( \cos\left(\frac{1}{x}\right) \right)^{x^2}$ .



## Exercise 2 (5 points)

1. Using d'Alembert's test, determine the nature of  $\sum \frac{(n!)^3}{(3n)!}$ .

2. Using a Taylor expansion, determine the nature of  $\sum \ln\left(\cos\left(\frac{1}{n}\right)\right)$ .

3. Determine the nature of  $\sum n^2 e^{-n}$ .

4. Determine the nature of  $\sum \frac{(-1)^n}{n\sqrt{n}}$ .

### Exercise 3 (4 points)

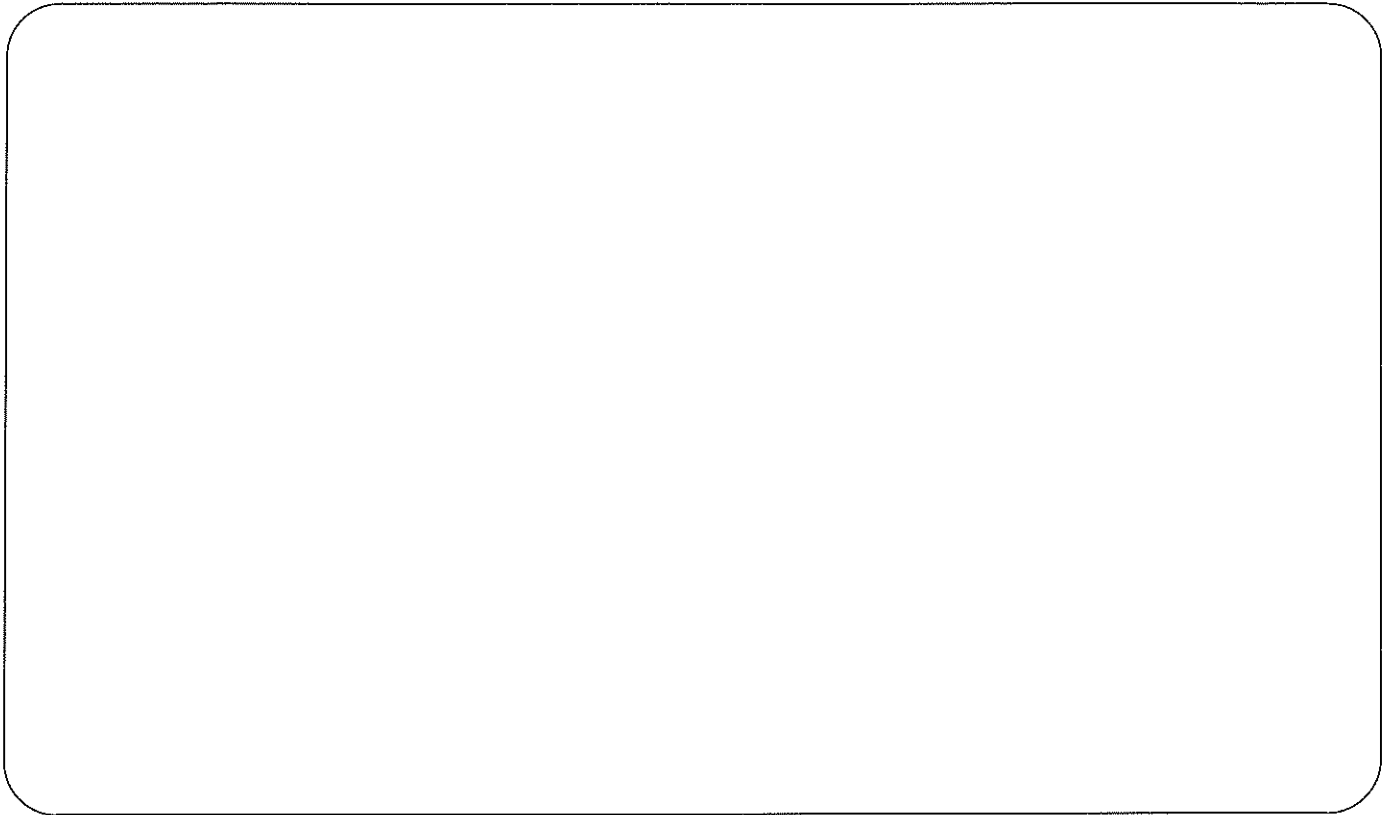
Let us consider the sequence  $(u_n)_{n \in \mathbb{N}^*}$  defined for every  $n \in \mathbb{N}^*$  by

$$u_n = \ln((n-1)!) - \left(n - \frac{1}{2}\right) \ln(n) + n$$

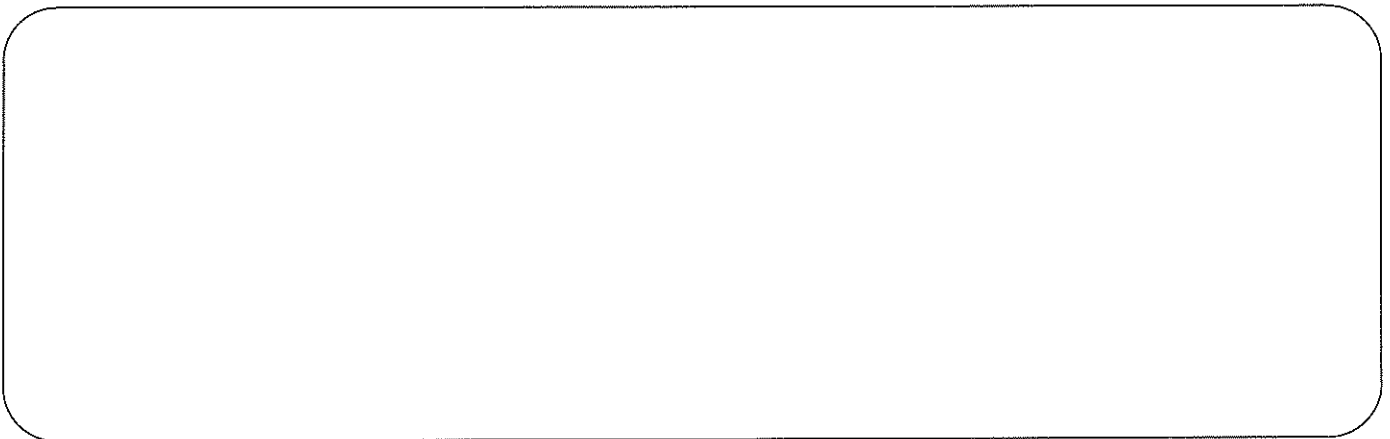
1. Show that

$$u_{n+1} - u_n = 1 - \left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right)$$

2. Show that  $u_{n+1} - u_n \underset{+\infty}{\sim} -\frac{1}{12n^2}$ .



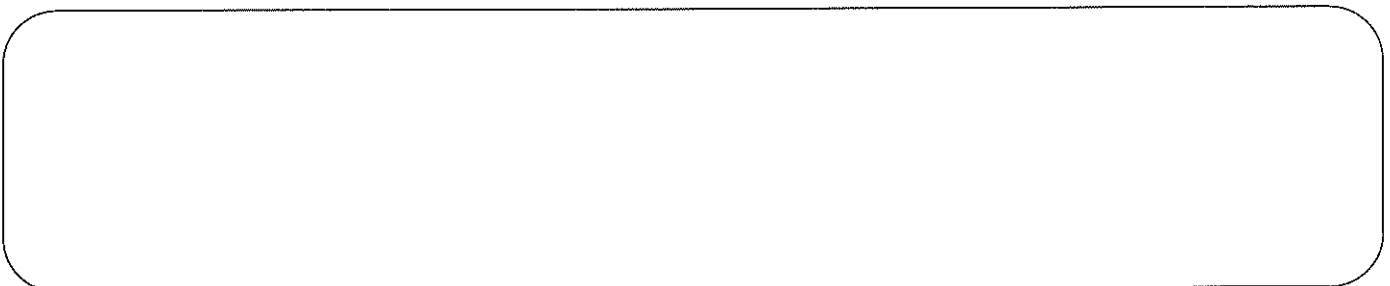
3. Deduce that  $(u_n)$  is convergent.



### Exercise 4 (4 points)

Let us consider the sequence  $(u_n)$  defined for every  $n \in \mathbb{N}^*$  by  $u_n = \sqrt[n+1]{n+1} - \sqrt[n]{n}$ .

1. Determine  $\lim_{n \rightarrow +\infty} \sqrt[n]{n}$ .



2. Show that for every  $n \in \mathbb{N}^*$ ,  $u_n = \sqrt[n]{\left(1 + \frac{1}{n}\right)^{1/n} - 1}$ .

3. Using a Taylor expansion, determine an equivalent of  $\left(1 + \frac{1}{n}\right)^{1/n} - 1$ , then of  $u_n$ .

4. Deduce the nature of  $\sum u_n$ .

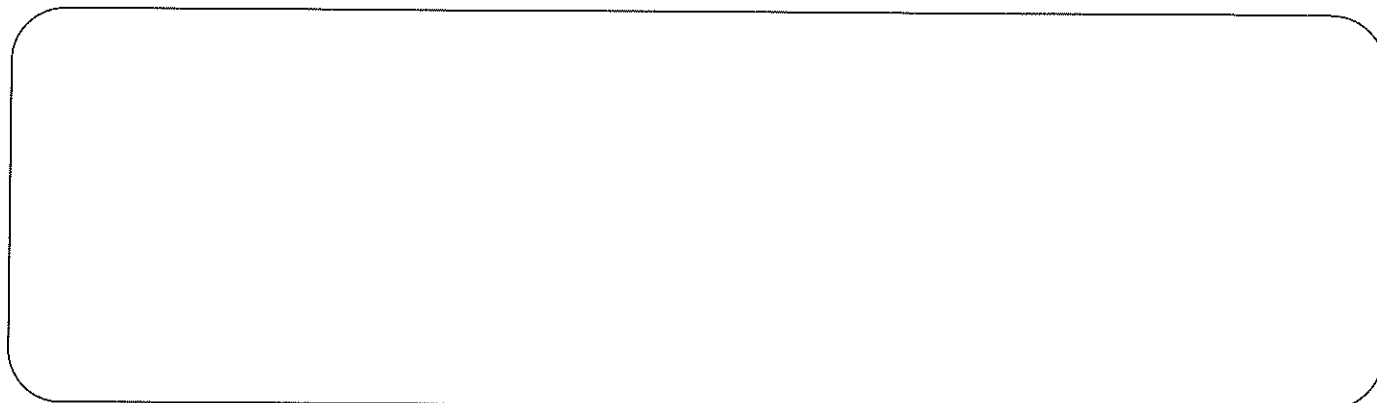
### Exercise 5 (4 points)

Let us consider the sequence  $(u_n)$  defined for every  $n \geq 2$  by  $u_n = \frac{(-1)^n}{\ln(n) - (-1)^n}$ .

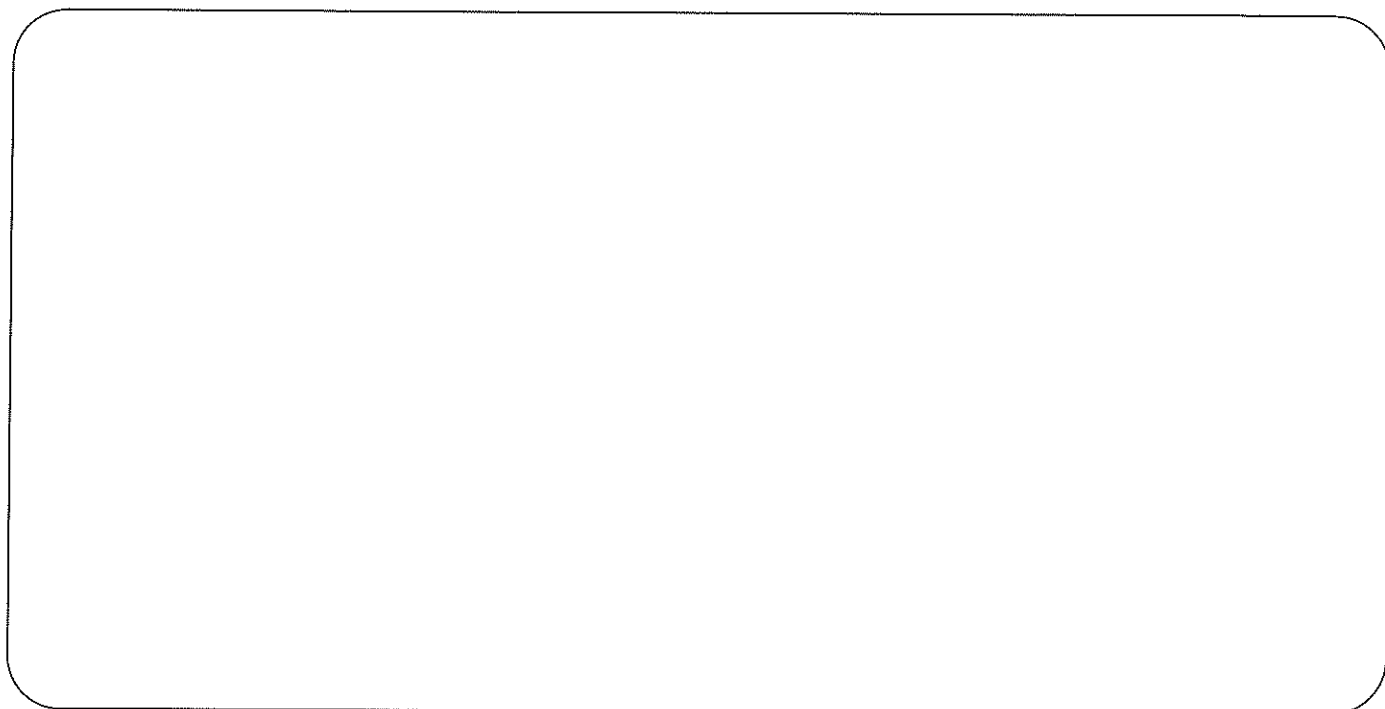
1. What is the following limit :  $\lim_{n \rightarrow +\infty} \frac{n}{\ln^2(n)}$  ?

2. Deduce the nature of  $\sum \frac{1}{\ln^2(n)}$ .

3. Check that for every  $n \in \mathbb{N}^*$ ,  $u_n = \frac{(-1)^n}{\ln(n)} \left(1 - \frac{(-1)^n}{\ln(n)}\right)^{-1}$ .



4. Determine  $a \in \mathbb{R}$  such that  $u_n = \frac{(-1)^n}{\ln(n)} + \frac{a}{\ln^2(n)} + o\left(\frac{1}{\ln^2(n)}\right)$ .



5. Deduce the nature of  $\sum u_n$ .

