EPITA / S3 December [redacted]

NAME : .... FIRSTNAME : .... GROUP : ....

Physics S3 Final

The calculators and the extra-documents are not allowed.

Answer only on the exam sheet.

Remember that, except if explicitly written in the questions, the notation $E_A(M)$ corresponds to the **norm** of the field $\overrightarrow{E_A}(M)$. But the angles are **oriented.**

We will use for convenience the usual constant $k = \frac{1}{4\pi\epsilon_0}$.

<u>MCQ</u> ***(4 points-no negative points)*** ***Circle the right answer.***

1- The electric field, generated by a pointlike charge q located at point O, at any point M reads:

a) $\vec{E}(M) = k\frac{q}{OM^3}\overrightarrow{OM}$ b) $\vec{E}(M) = k\frac{q}{OM^2}\overrightarrow{OM}$ c) $\vec{E}(M) = k\frac{q}{OM}\overrightarrow{OM}$

2- A charge q is located in an electric field $\vec{E}$. The force $\vec{F}$, which is acting on the charge q, is given by:

a) $\vec{F} = -q.\vec{E}$ b) $\vec{F} = |q|.\vec{E}$ c) $\vec{F} = q.\vec{E}$

3- The electric field created by a positive charge located at O is:

a) Convergent b) Well-defined at O c) Divergent

4- Which property satisfies the electrostatic field $\vec{E}$ related to the potential V ?

a) $\vec{E} = \overrightarrow{grad}(V)$ b) $\vec{E} = -\overrightarrow{grad}(V)$ c) $V = \overrightarrow{grad}(\vec{E})$

5- The area of a sphere of radius R is:

a) $4\pi R^3$ b) $\frac{4}{3}\pi R^2$ c) $4\pi R^2$

6- Let us consider a volume $\mathcal{V}$ containing a charge Q_{int} and delimited by a surface $\mathcal{S}$. The Gauss theorem reads for the field $\vec{E}$ created by this geometry:

a) $\oiint_S \vec{E}.\overrightarrow{dS} = \frac{Q_{int}}{\epsilon_0}$ b) $\oiint_S \vec{E}.\overrightarrow{dS} = Q_{int}$ c) $\oiint_S E\,dS = \frac{Q_{int}}{\epsilon_0}$

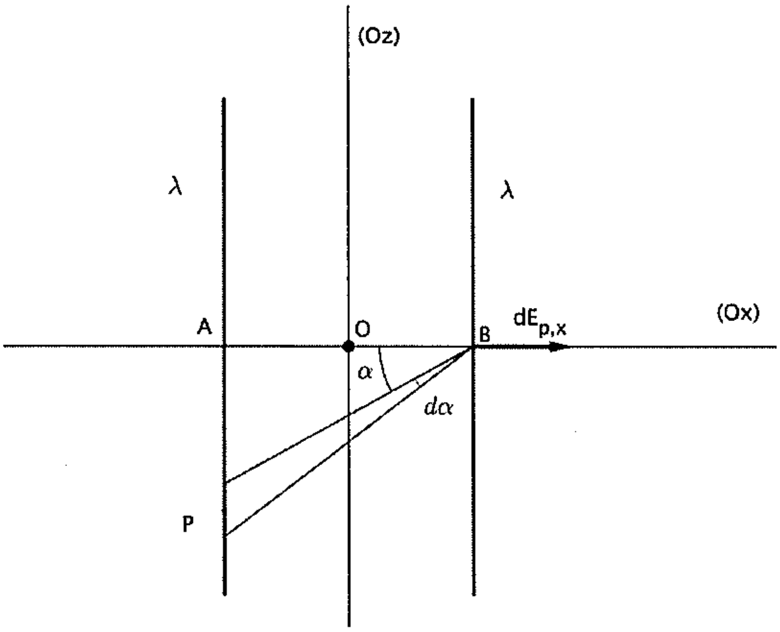
7- We consider the limit case of an infinite cylinder of axis (Oz) and radius R. This cylinder is uniformly charged on its surface. What can be claimed at any $M(r < R)$ inside the cylinder?

a) $\vec{E}(M) = \vec{0}$ b) $E(M) = E_0 \ln\left(\frac{r}{R}\right)$ c) $E(M) = E_0 R^2/r^2$

8- At some point M the charge distribution has a symmetry plane $\mathcal{P}$. Therefore:

a) $\vec{E}(M) \perp \mathcal{P}$ b) $\vec{E}(M) \in \mathcal{P}$ c) $\vec{E}(M) \notin \mathcal{P}$ yet $\vec{E}(M) \parallel \mathcal{P}$

Exercise 1 (*4 points*)



Two wires of finite length $2l$ with a lineic charge distribution λ are separated by a distance l, as sketched above. We assume that the axis (Oz) is a symmetry axis of the distribution.

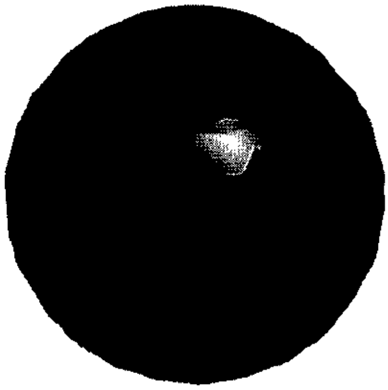
1- a) Express the total electric field $\vec{E}(B)$ created at B by the wire containing A. Remember that the component along $\overrightarrow{u_x}$ of the elementary electric field created by a length element centered at P and located at an angle α can be written as: $dE_{p,x}(B) = \frac{k\lambda}{l}\cos\alpha\, d\alpha$. Deduce its norm.

b) By explaining your answer, give the expression of the corresponding electric field created at A by the wire containing B.

2- a) The wire containing A is modelled as a pointlike Q_A located at A. Express the Q_A in terms of l and λ. Write then its potential electrostatic energy $\mathcal{E}_A$.

b) Deduce the total potential electrostatic energy of the two wires.

Exercise 2 *(6 points)*



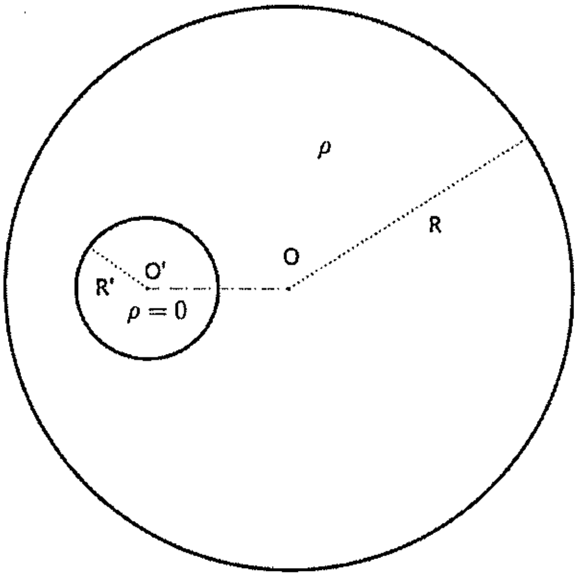
Consider a volumic positive charge distribution ρ, uniformly distributed in a ball of center O and radius R, denoted by $\mathcal{B}(O, R)$.

1- Study the invariances and the symmetries of the charge distribution. Deduce the form of the electric field $\vec{E}(M)$ at any point M.

2- By using the Gauss theorem, express the electric $\vec{E}(M)$, for M inside and outside the ball.

3- Deduce the electrostatic potential $V(M)$ at any point M. Draw the curve representing this function.

4- Consider now the following charge distribution:



For convenience, the former ball is sketched along a transversal cut. There is clearly a charge depletion (locally the density is vanishing). That one is located in a ball of radius R' and center O' : $\mathcal{B}(O', R')$, with O' in the ball $\mathcal{B}(O, R)$ and $R' < R$ (see above).

a) Using the question 2, deduce the expression of the electric field $\vec{E}'(M)$ generated by the ball $\mathcal{B}(O', R')$, which is uniformly charged with a density $-\rho$, at any point $M(r')$ for $r' < R'$ et $r' > R'$. Here $r' = O'M$.

b) Deduce the total field $\vec{E}_{tot}(M)$ created at any point M by the total distribution sketched above.

c) Describe the shape of the field $\vec{E}_{tot}(M)$ for M in the cavity.

Exercise 3 *(6 points)*

In a cylindrical conductor of axis (Oz), of radius R and of length l, flows a current density $\vec{J}$ given by $\vec{J}(r) = J_0\left(1 - \frac{r^2}{R^2}\right)\vec{u}_z$. R and J_0 are constants. The conductivity of this conductor is denoted by σ.

1- Express the current $I(r)$ through a section of radius r and of normal vector $\vec{u}_z$. Deduce the total current I in this conductor.

2- Recall the expression of the local Ohm law. Deduce the expression of the electric $\vec{E}$ in this conductor.

3- We consider a uniform current $I_{tot} = \frac{\pi R^2 J_0}{2}$ in this conductor of section S and length l. Determine:

a) Its resistance R.

b) Its voltage U.

c) The expression of the mean speed of the electrons v_{e^-} in terms of I_{tot}. The electrons density is denoted by n_{e^-} and the current density is given by $\vec{J} = -e.n_{e^-}.\ \overrightarrow{v_{e^-}}$

Appendix: brief clarification and mathematical reminders

Exercise 1

The axes (Ox) and (Oz) are symmetries axes of the charge distribution.

Remember that the potential energy of a system associated to a force $\vec{F}$, conservative by definition, can be derived using the elementary work $\delta W(\vec{F}) = \vec{F}.\overrightarrow{dl} = -d\mathcal{E}$.

Dans le cadre de cet exercice, en électrostatique, on utilisera $\overrightarrow{dl} = dl\,\overrightarrow{u_x}$ où dl correspond à une variation infinitésimale de la longueur l.

Exercise 2

In the specific case of a discreet charge distribution $\{q_i\}$, generating the corresponding electric fields $\{\overrightarrow{E_i}\}$, the total field can be seen as their superposition, i.e. $\overrightarrow{E_{tot}} = \sum_i \overrightarrow{E_i}$.

We have seen, in the course on the continuous charge distributions, that this formula can be generalized. For a volumic charge distribution ρ defined in $\mathcal{V}$, we know that:

$$\overrightarrow{E_{tot}}(M) = \iiint_{P\in\mathcal{V}} d\overrightarrow{E_P}(M)$$

It can be relevant to decompose this integral in a sum of two integrals. For instance, for a x-variable function f, defined and continuous on the interval $[a, b]$, $a < b$:

$$\forall c \in [a, b], \int_c^b f(x)dx = \int_a^b f(x)dx - \int_a^c f(x)dx$$

For the total field $\overrightarrow{E_{tot}}$ (question 4-b)), express the result in terms of r, r', ρ and the vector $\overrightarrow{OO'}$.

Exercise 3

To avoid misunderstanding and confusion with the resistance R, consider the radius of the conductor as being a_0 and the following current density $\vec{J}(r) = J_0\left(1 - \frac{r^2}{a_0^2}\right)\vec{u}_z$.

Idem for the last part of the exercise, you can replace the radius in $I_{tot} = \frac{\pi a_0^2 J_0}{2}$ (keep the letter R for the resistance).