

Midterm exam of physics

*Calculators and documents are not allowed. The number of points per question is indicative.
Answers to be written on this document only. Useful formulas are given in the annex*

Exercise 1 (5 points)

The questions a and b are independent.

a- Let consider $T(x, y, z)$ a temperature scalar function. $T(x, y, z)$ is a total differential, prove that:

$$\vec{\text{curl}}(\vec{\text{grad}}(T)) = \vec{0}$$

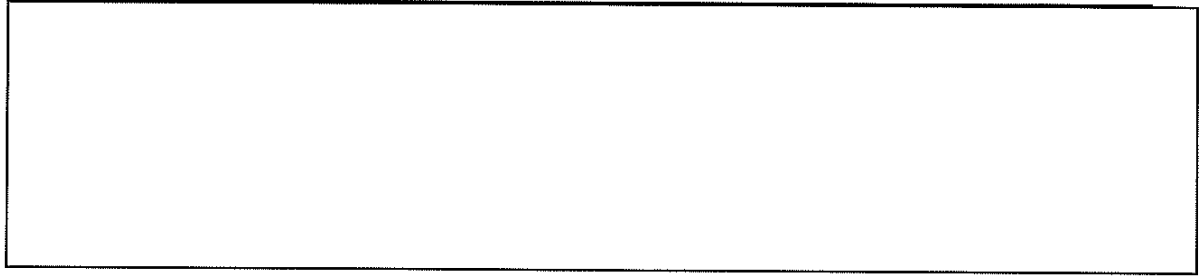
b- A spherical distribution of electric charges creates an electric potential $V(r)$. Its expression at a point M of space is given by: $V(r) = K.r \exp(-\alpha.r)$; where α and K are constant.

Using spherical coordinates:

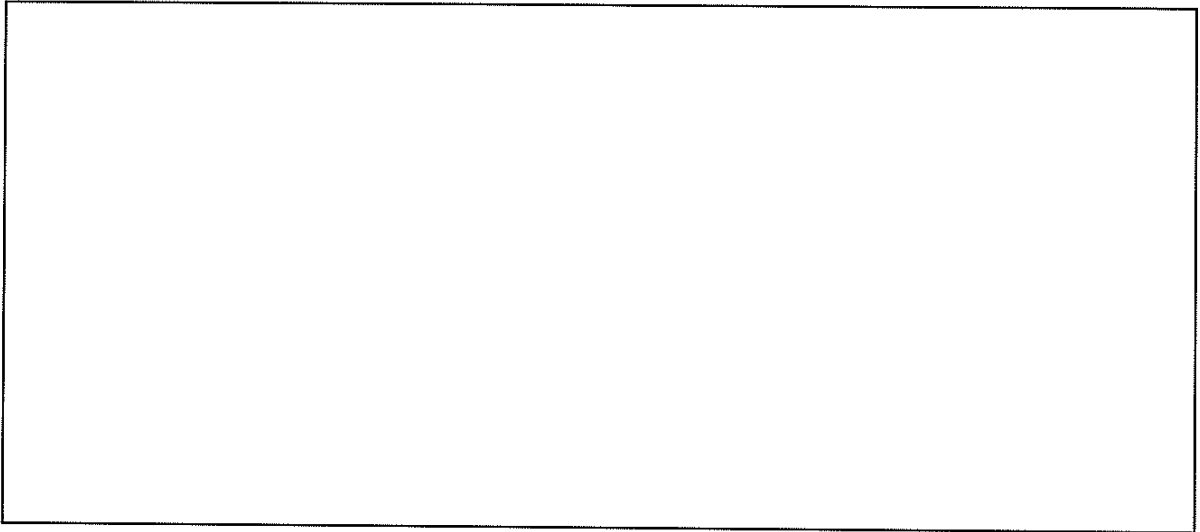
$$\vec{\text{grad}}(f(r, \theta, \varphi)) = \frac{\partial f}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \vec{e}_\varphi$$

Where $f(r, \theta, \varphi)$ is a given function.

1- Without doing calculation, give the direction of the electric field vector \vec{E} , knowing that:
 $\vec{E} = -\vec{\text{grad}}(V)$.



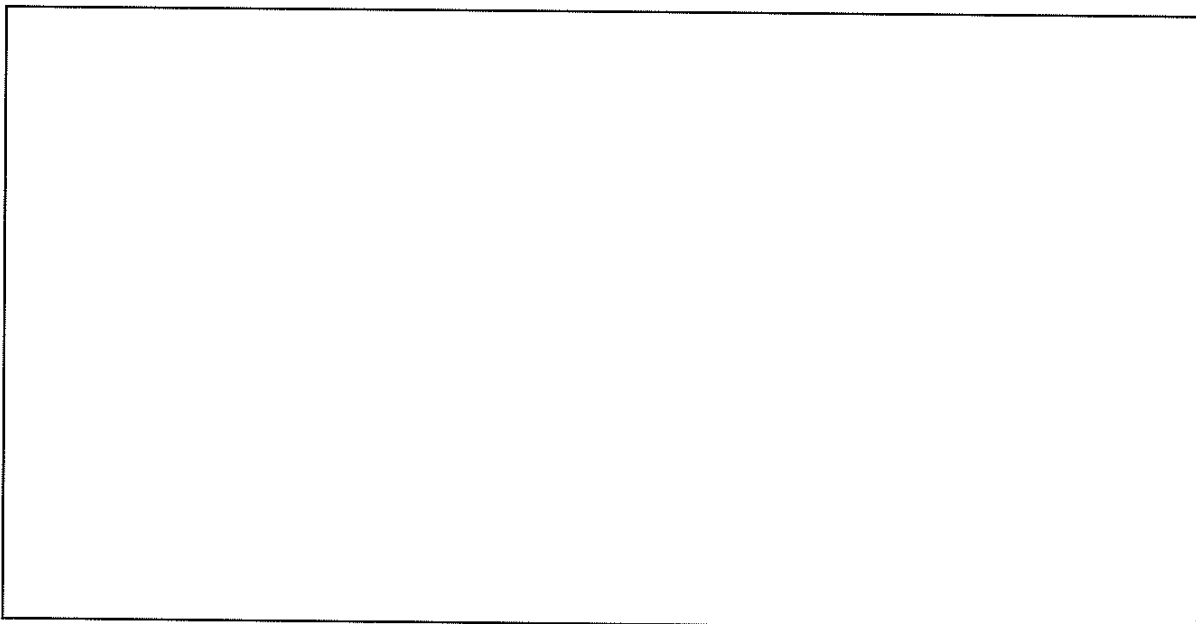
- 2- Use the previous formula to express the components of the electric field vector \vec{E} created by the spherical distribution.



Exercise 2 (8 points)

The questions I, II and III are independent.

- I-a) Do the demonstration of the second Maxwell's equation.



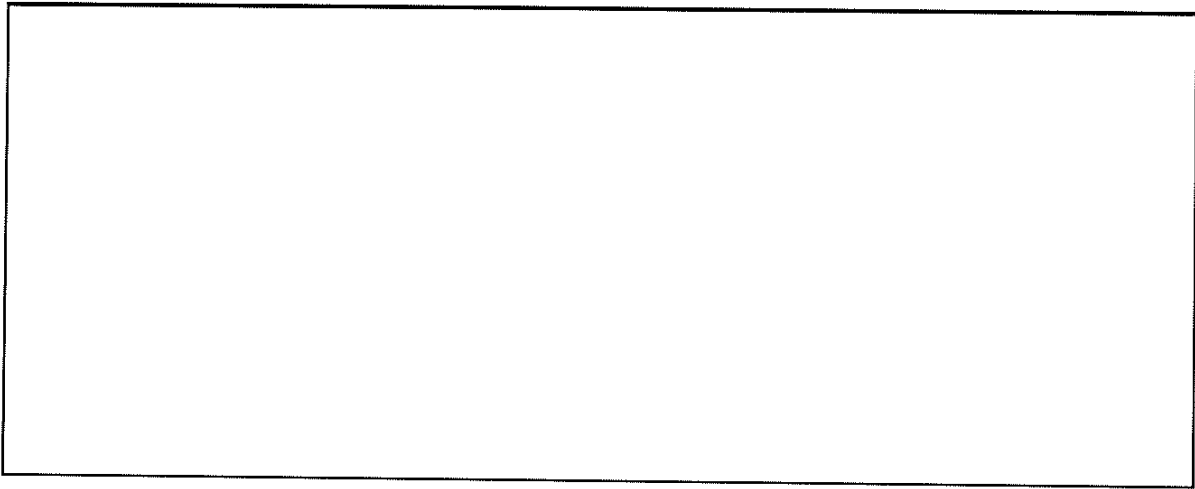
I- b) Give the meaning of the second Maxwell's equation.

II) Do the demonstration of the first Maxwell's equation.

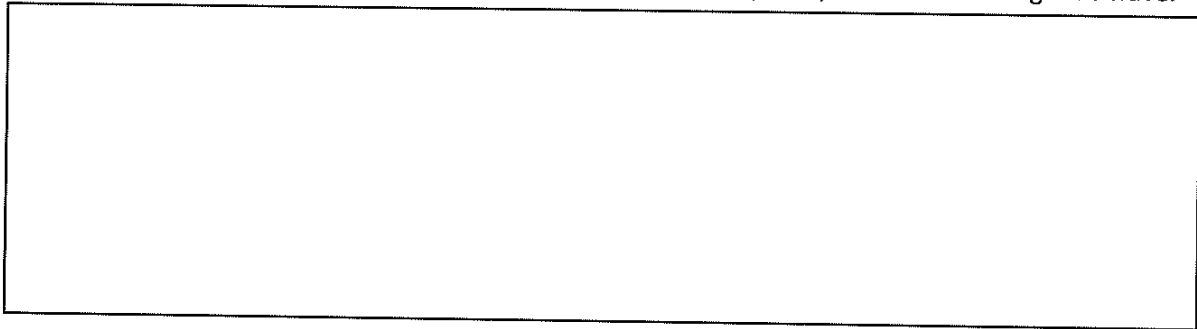
III- a) Using the Maxwell equations, find the propagation equation of the magnetic field in any material medium, given by:

$$\Delta \vec{B} - \mu \cdot \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu \cdot \overrightarrow{\text{curl}}(\vec{J}).$$

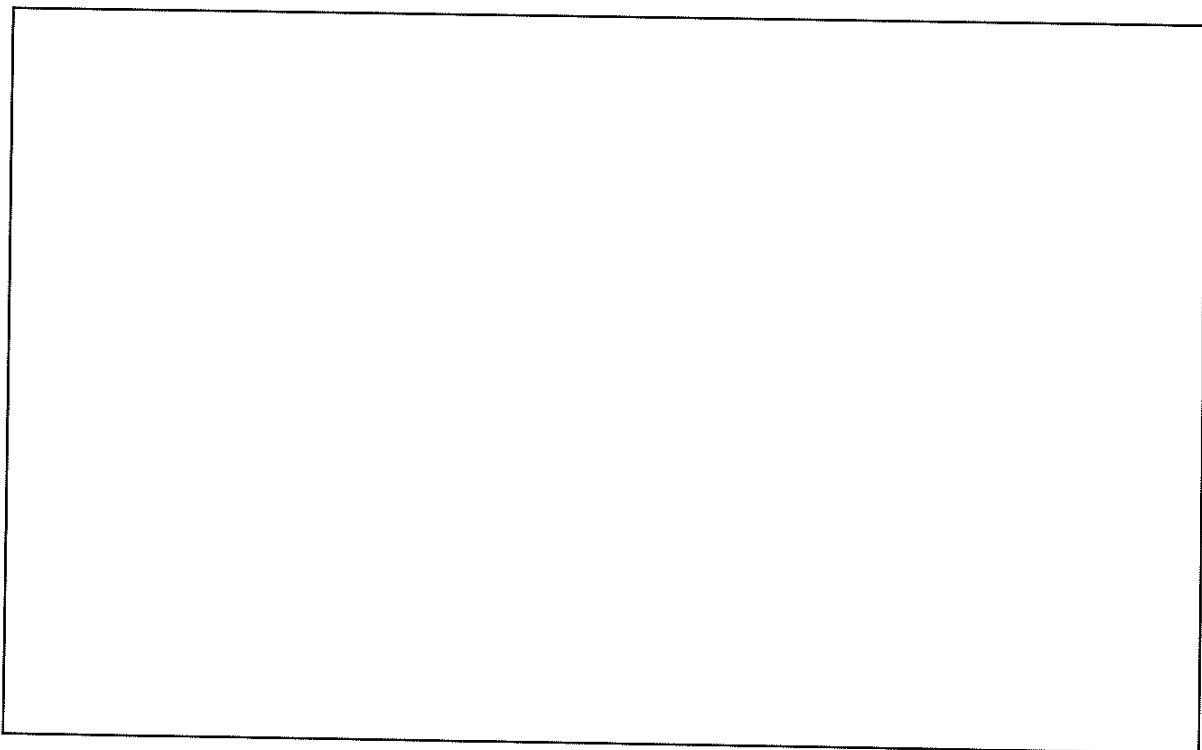
Given: $\Delta \vec{U} = \overrightarrow{\text{grad}}(\text{div}(\vec{U})) - \overrightarrow{\text{curl}}(\overrightarrow{\text{curl}}(\vec{U}))$ for a given vector \vec{U} .

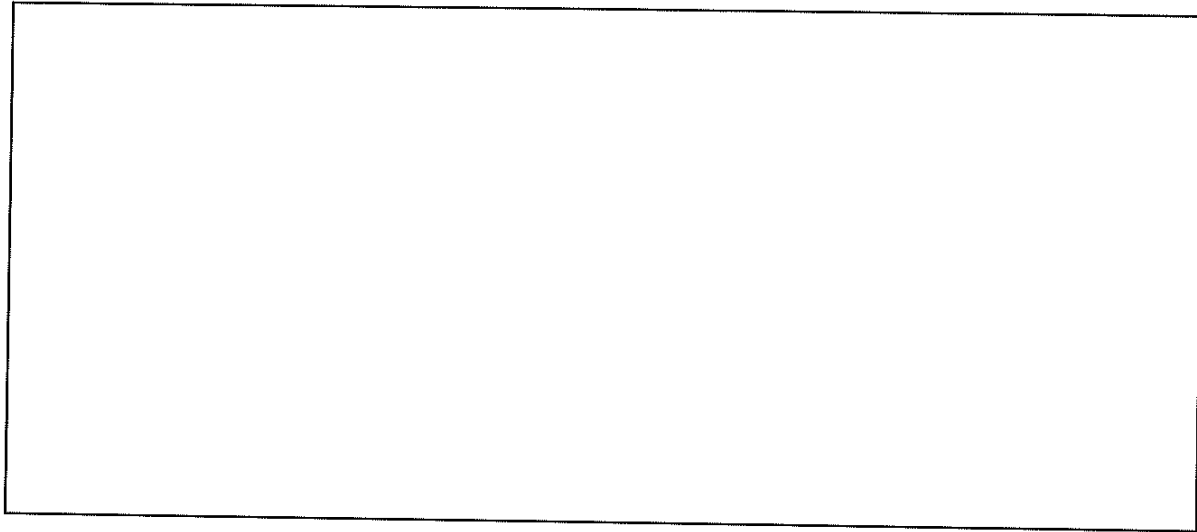


III-a) Give the propagation equation of the magnetic field in the air medium (or vacuum). Give the meaning of the constant $\mu_0 \cdot \epsilon_0$. Deduce the velocity in vacuum (or air) of the electromagnetic wave.



III-c) We know that the magnetic field given by : $\vec{B}(y,t) = B_0 \cos(k \cdot y - \omega t) \vec{e}_z$ is a solution of the propagation equation in the air medium, find a relationship between ω and k .

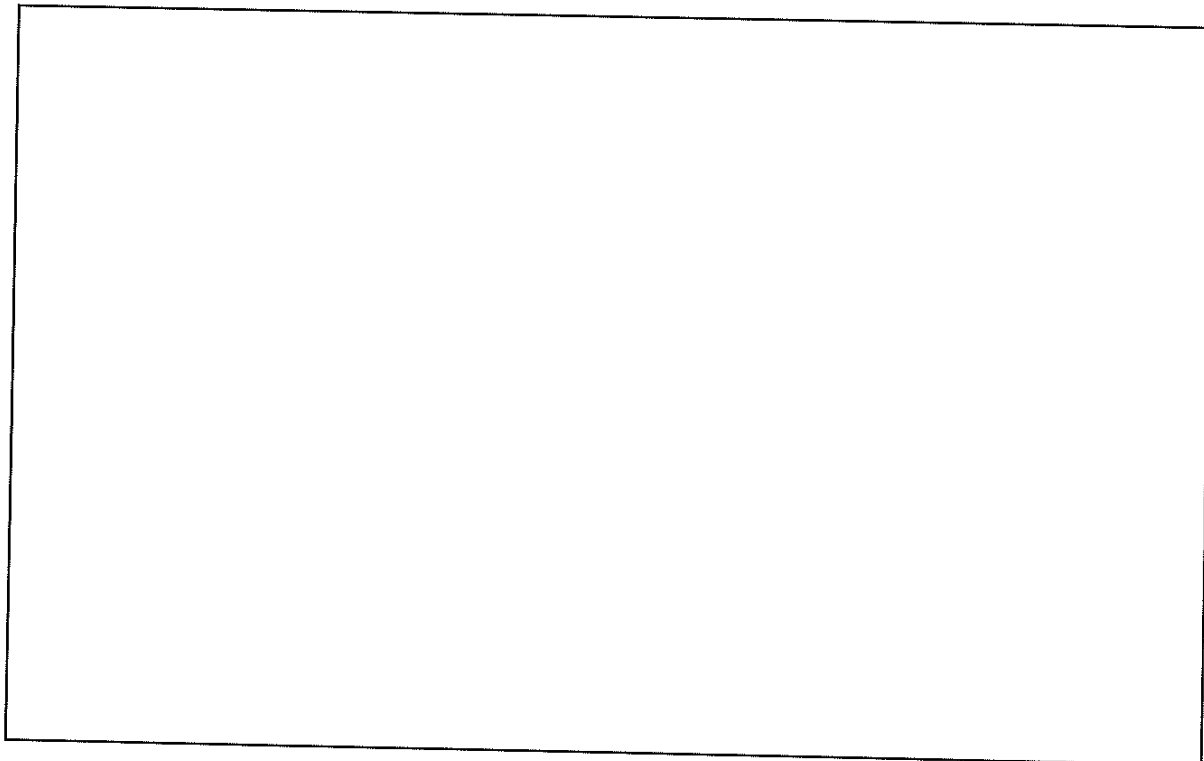


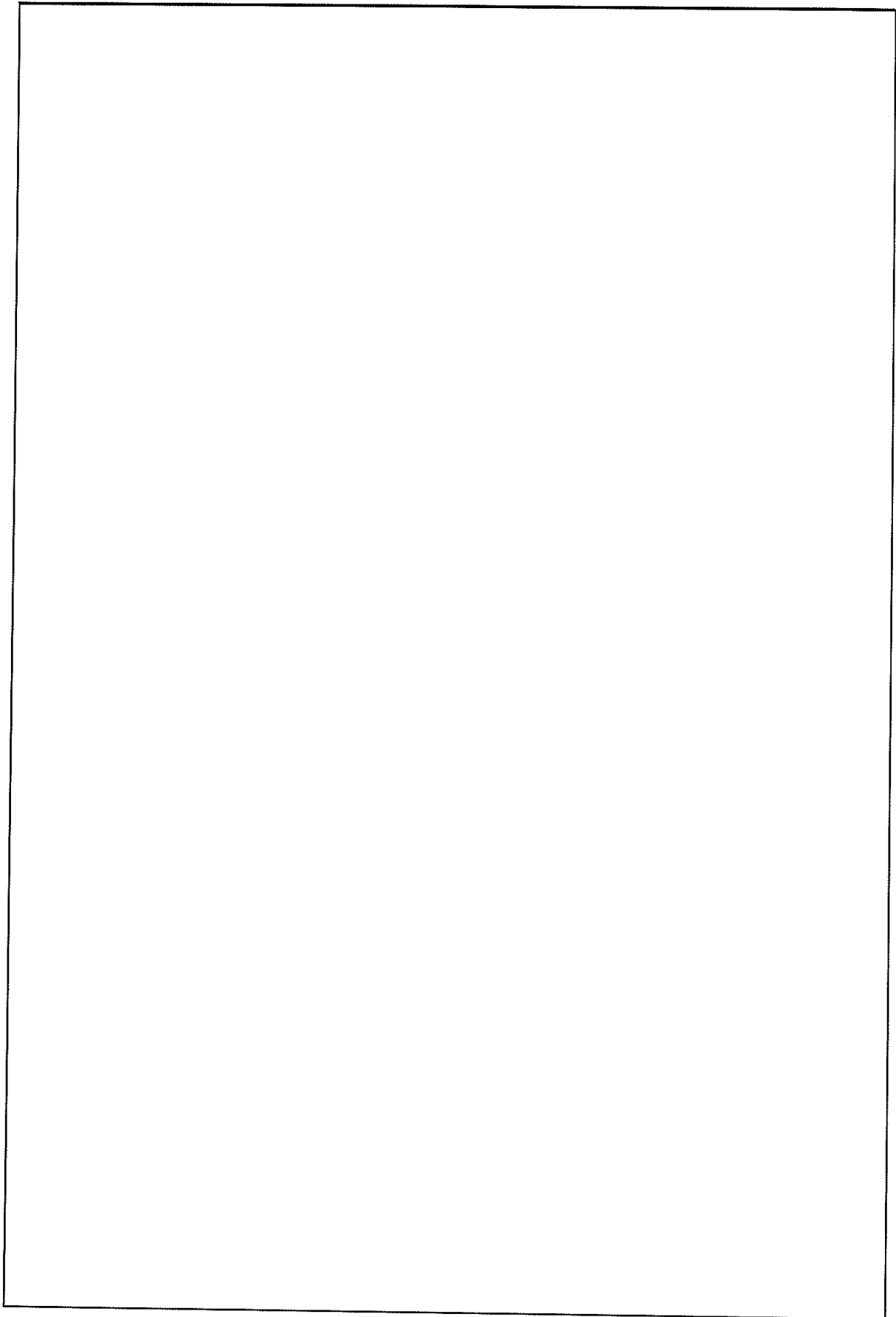
**Exercise 3** (7 points)

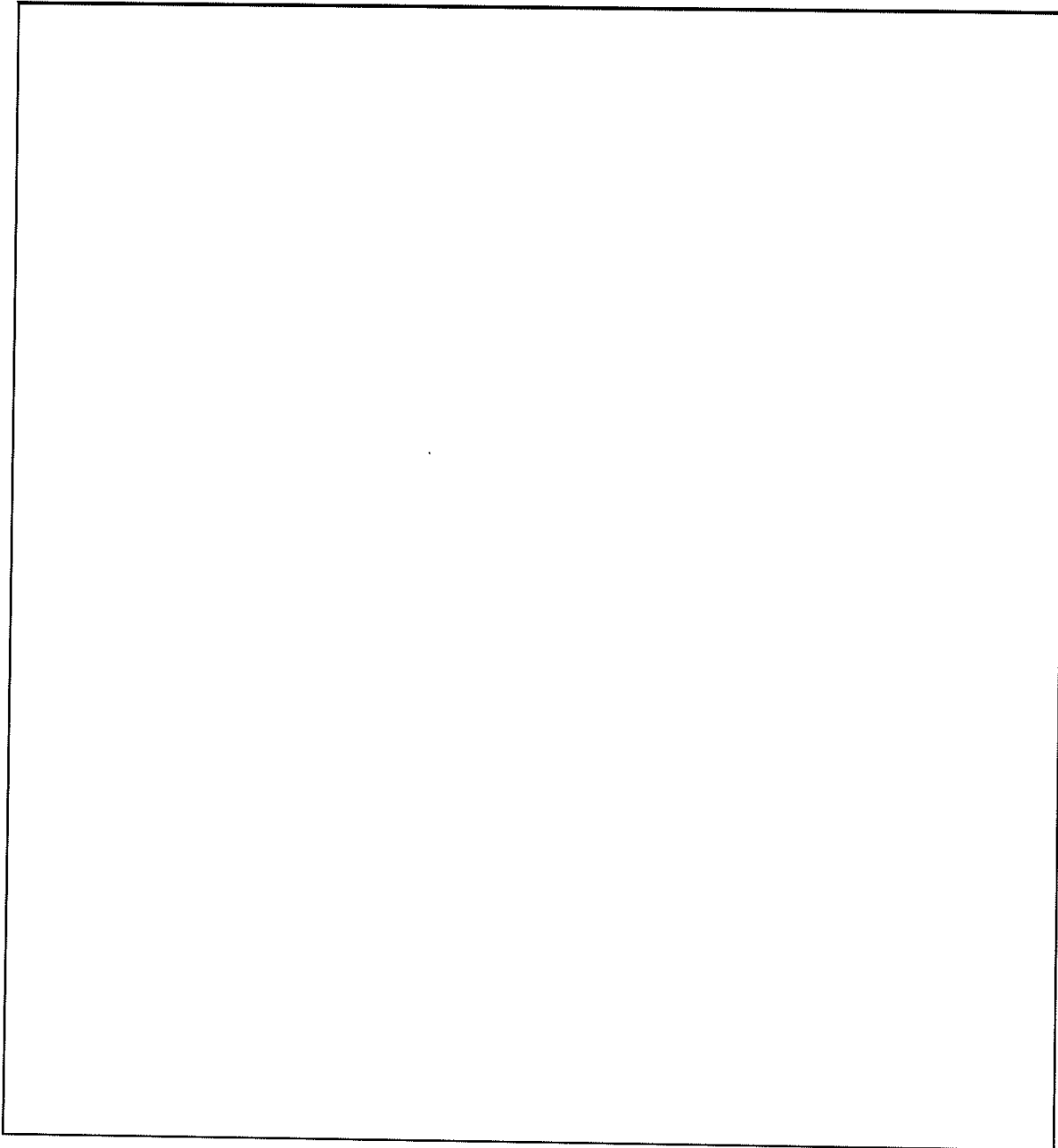
The electromagnetic field vectors of a sinusoidal progressive plane wave propagating in the air medium are given by:

$$\begin{cases} \vec{E}(x,t) = E_0 \cos(k.x - \omega.t) \vec{e}_y, \\ \vec{B}(x,t) = \frac{E_0}{c} \cos(k.x - \omega.t) \vec{e}_z. \end{cases}$$

Prove that these vectors satisfy the four Maxwell equations in the air environment. We know that:
 $\omega = k.c.$







Useful formulas

Maxwell equations in any medium:

$$1) \operatorname{div}(\vec{E}) = \frac{\rho}{\epsilon}$$

$$3) \operatorname{curl}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$$

$$2) \operatorname{div}(\vec{B}) = 0$$

$$4) \operatorname{curl}(\vec{B}) = \mu \cdot \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

The Green-Ostrogradski theorem: $\oiint_S \vec{U} \cdot d\vec{S} = \iiint_V \operatorname{div}(\vec{U}) d\tau$