EPITA / S3 December 2015

NAME : .... *First name:*.. GROUP :.........

Midterm exam of physics

Calculators and documents are not allowed. The number of points per question is indicative.
Answers to be written on this document only. Useful formulas are given in the annex

Exercise 1 (5 points)

The questions a and b are independent.

a- Let consider $T(x,y,z)$ a temperature scalar function. $T(x,y,z)$ is a total differential, prove that:

$$\overrightarrow{curl}(grad\vec{d}(T)) = \vec{0}$$

b- A spherical distribution of electric charges creates an electric potential V(r). Its expression at a point M of space is given by: $V(r) = K.r\exp(-\alpha.r)$; where α and K are constant.
Using spherical coordinates:

$$\overrightarrow{grad}(f(r,\theta,\varphi)) = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r}\sin\theta\frac{\partial f}{\partial \varphi}\vec{e}_\varphi$$

Where f (r, θ , φ) is a given function.

1- Without doing calculation, give the direction of the electric field vector $\vec{E}$, knowing that: $\vec{E} = -gra\vec{d}(V)$.

2- Use the previous formula to express the components of the electric field vector $\vec{E}$ created by the spherical distribution.

Exercise 2 (8 points)

The questions I, II and III are independent.

I-a) Do the demonstration of the second Maxwell's equation.

I- b) Give the meaning of the second Maxwell's equation.

II) Do the demonstration of the first Maxwell's equation.

III- a) Using the Maxwell equations, find the propagation equation of the magnetic field in any material medium, given by:

$$\Delta\vec{B} - \mu.\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} = -\mu.\overrightarrow{curl}(\vec{J}).$$

Given: $\Delta\vec{U} = \overrightarrow{grad}(div(\vec{U})) - \overrightarrow{curl}(\overrightarrow{curl}(\vec{U}))$ for a given vector $\vec{U}$.

III-b) Give the propagation equation of the magnetic field in the air medium (or vacuum). Give the meaning of the constant $\mu_0.\varepsilon_0$. Deduce the velocity in vacuum (or air) of the electromagnetic wave.

III-c) We know that the magnetic field given by : $\vec{B}(y,t) = B_0 \cos(k.y - \omega.t)\vec{e}_z$ is a solution of the propagation equation in the air medium, find a relationship between ω and k.

Exercise 3 (7 points)

The electromagnetic field vectors of a sinusoidal progressive plane wave propagating in the air medium are given by:

$$\begin{cases} \vec{E}(x,t) = E_0 \cos(k.x - \omega.t)\vec{e}_y \\ \vec{B}(x,t) = \dfrac{E_0}{c} \cos(k.x - \omega.t)\vec{e}_z \end{cases}$$

Prove that these vectors satisfy the four Maxwell equations in the air environment. We know that:

ω = k.c.

Useful formulas

Maxwell equations in any medium:

1) $div(\vec{E}) = \dfrac{\rho}{\varepsilon}$

2) $div(\vec{B}) = 0$

3) $\overrightarrow{curl}(\vec{E}) = -\dfrac{\partial \vec{B}}{\partial t}$

4) $\overrightarrow{curl}(\vec{B}) = \mu.\vec{J} + \mu\varepsilon\dfrac{\partial \vec{E}}{\partial t}$

The Green-Ostrogradski theorem: $\oiint_S \vec{U}.d\vec{S} = \iiint_\tau div(\vec{U})d\tau$