EPITA

Mathematics

Final exam (S3)

December 2018

Name:		
First name :		
Class:		

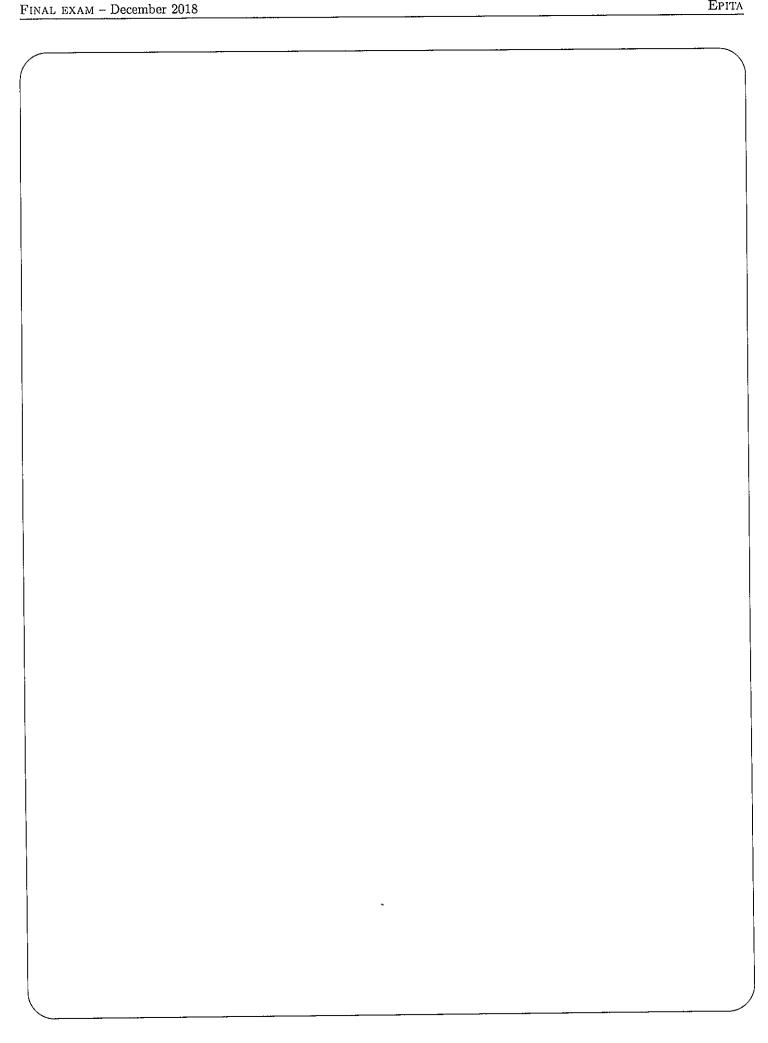
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Exercise 1 (5 points)

Let $A = \begin{pmatrix} -1 & 2 & 1 \\ 2 & -1 & -1 \\ -4 & 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & -1 \\ -3 & 4 & -3 \\ -1 & 1 & 0 \end{pmatrix}$.

Are A and B diagonalizable in $\mathcal{M}_3(\mathbb{R})$? If they are, determine D and P.

N.B.: the bases of the eigenspaces must be deduced from a clear reasoning, and not by randomly picking particular values.

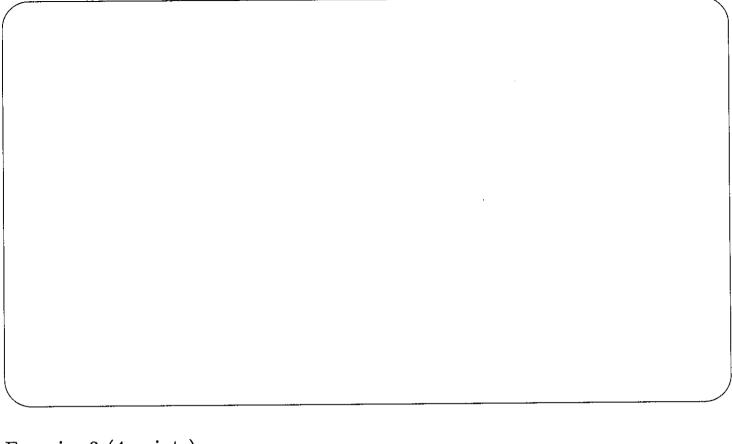


Exercise 2 (3 points)

Let $a \in \mathbb{R}$ and $A = \begin{pmatrix} 3-a & -5+a & a \\ -a & a-2 & a \\ 5 & -5 & -2 \end{pmatrix}$.

Study the diagonalizability of A in $\mathcal{M}_3(\mathbb{R})$ depending on the value of a.

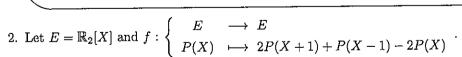
N.B. : When A is diagonalizable, the eigenbasis is not required.



Exercise 3 (4 points)

1. Let $f: \left\{ \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R}^2 \\ (u, v, w) & \longmapsto & (2u - w; 3u + v + 2w) \end{array} \right.$

Determine the matrix of f in the standard bases of the input and output spaces.



Determine the matrix of f in the standard basis $(1, X, X^2)$ of $\mathbb{R}_2[X]$.

Exercise 4 (4 points)

Let E and F be two vector spaces over \mathbb{R} , $f \in \mathcal{L}(E,F)$ and let $X = (x_1, \dots, x_n)$ be a family of vectors of E. Show that :

1. $f(\operatorname{Span}(X)) = \operatorname{Span}(f(X))$.

2. $[f \text{ surjective and } \operatorname{Span}(X) = E] \Longrightarrow \operatorname{Span}(f(X)) = F.$

3. $[f \text{ injective and } X \text{ linearly independent}] \Longrightarrow f(X) \text{ linearly independent}.$

Exercise 5 (3 points)

1. Prove that any polynomial of odd degree with real coefficients has at least one real root.

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2.	Let $A \in$	$\in \mathscr{M}_n(\mathbb{K})$	such that	A^2+A	A + I =	. U ((*J.	

a. Let $\lambda \in \operatorname{Sp}_{\mathbb{R}}(A)$. Show that $\lambda^2 + \lambda + 1 = 0$.

b. Show that a matrix $A \in \mathcal{M}_3(\mathbb{R})$ cannot satisfy the equation (*).	

Exercise 6 (2 points) Let $n \in \mathbb{N}^*$, $(a_1, \ldots, a_n) \in \mathbb{R}^n$, $(b_1, \ldots, b_n) \in \mathbb{R}^n$. Compute the determinant of size $n+1: \Delta_n =$	$\begin{vmatrix} 1 \\ b_1 \end{vmatrix}$	$\frac{1}{a_1}$	a_1			$\begin{bmatrix} 1 \\ a_1 \end{bmatrix}$	
The state of the s	b_1	b_2	a_2			a_2	
Let $n \in \mathbb{N}^*$, $(a_1, \ldots, a_n) \in \mathbb{K}^n$, $(v_1, \ldots, v_n) \in \mathbb{K}^n$. Compute the determinant of size $n+1$. $\Delta_n =$:	:	•			; ;	[
	b_1	b_2		•••	b_n	a_n	