

EPITA  
Mathematics

Final exam (S2)

June, 2017

Name :

First name :

Class :

GRADE :



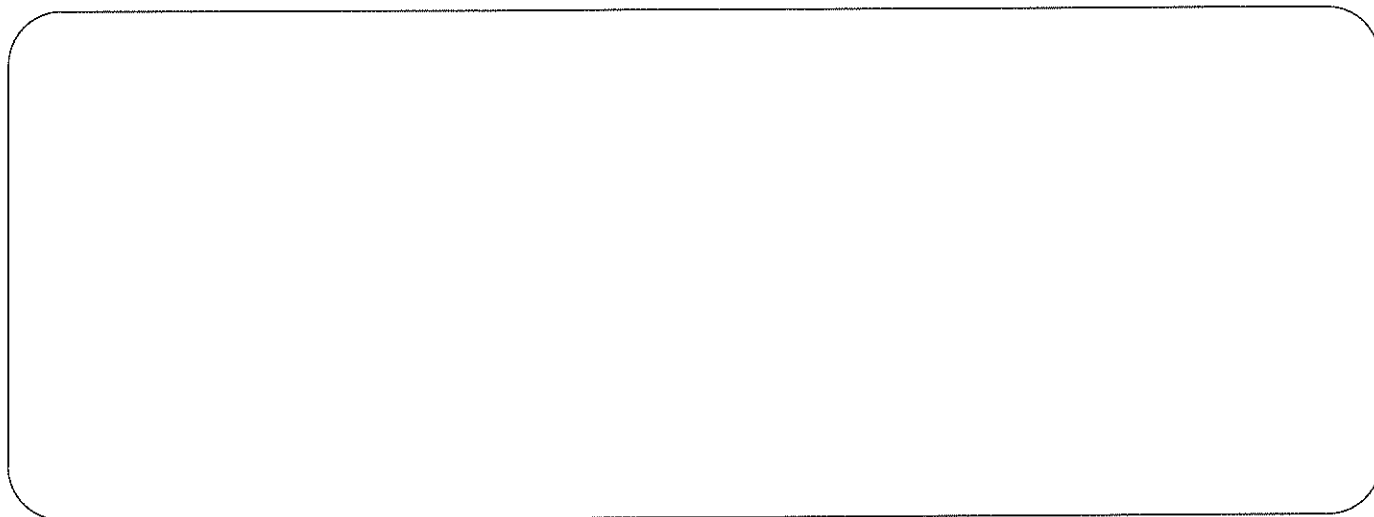
**Exercise 1 (2 points)**

Let  $A = \begin{pmatrix} -1 & 3 & -1 \\ 3 & -5 & 2 \\ -1 & 2 & -1 \end{pmatrix}$ . Determine the matrix  $A^{-1}$  (check the final result carefully on your draft).

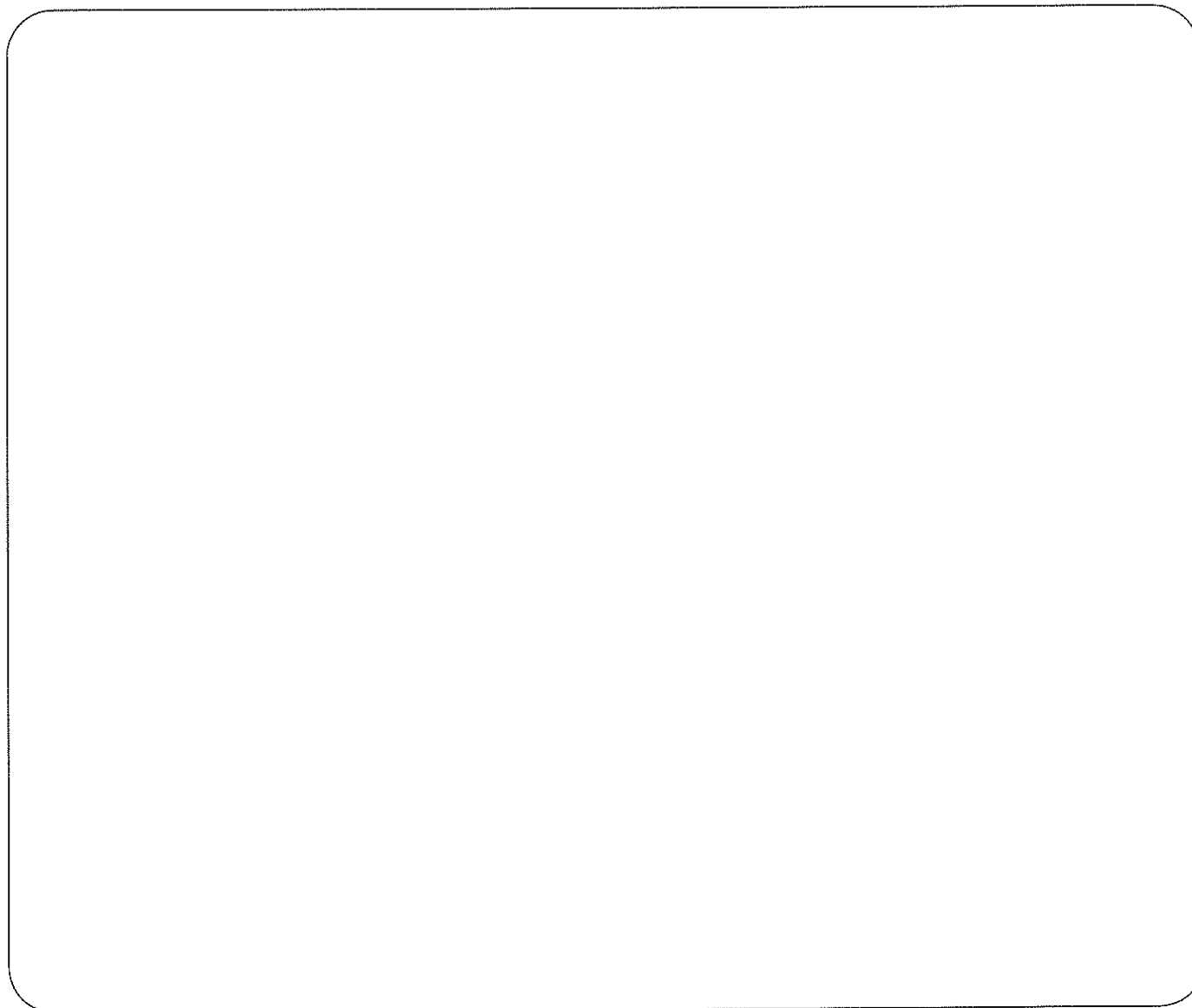
## Exercise 2 (4,5 points)

Expand into partial fractions of  $\mathbb{R}(X)$  the following rational fractions :

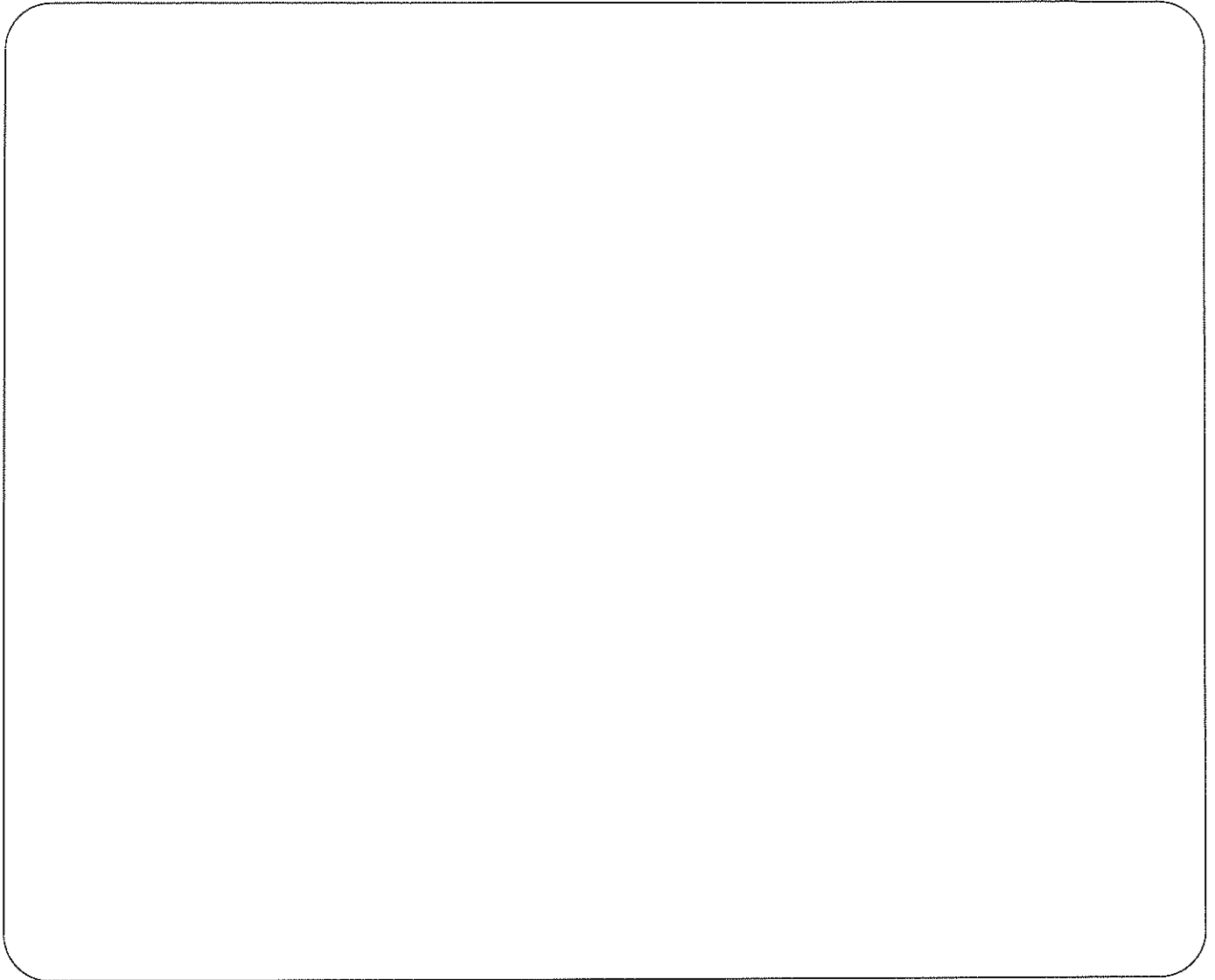
1. 
$$F(X) = \frac{X^2 + X + 1}{(X + 1)(X - 1)(X - 3)}$$



2. 
$$G(X) = \frac{X^3 - X - 1}{(X + 1)(X + 3)}$$



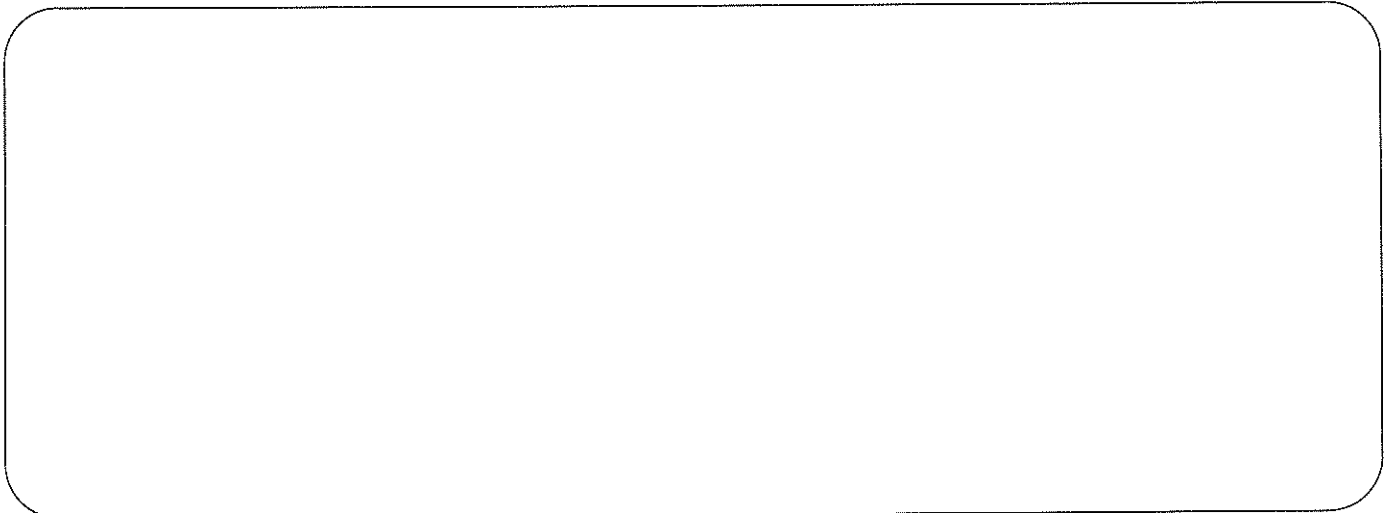
3.  $H(X) = \frac{X^2 - X - 1}{(X - 2)(X^2 + 1)}$



**Exercise 3 (3 points)**

1. Let  $f : \mathbb{R}_2[X] \longrightarrow \mathbb{R}^2$  be the linear map defined for any  $P \in \mathbb{R}_2[X]$  by  $f(P(X)) = (P(1), P(2))$ .

Determine the matrix of  $f$  in the standard bases of  $\mathbb{R}_2[X]$  and  $\mathbb{R}^2$ .



2. Let  $u : \begin{cases} \mathbb{R}^3 & \rightarrow \mathcal{M}_2(\mathbb{R}) \\ (x, y, z) & \mapsto \begin{pmatrix} x+y & y+z \\ x+z & 0 \end{pmatrix} \end{cases}$

Determine the matrix of  $u$  in the standard bases of  $\mathbb{R}^3$  and  $\mathcal{M}_2(\mathbb{R})$ .

### Exercise 4 (3 points)

Let  $E$  be a vector space over  $\mathbb{R}$  and  $f \in \mathcal{L}(E)$ . As usual, we denote  $f^2 = f \circ f$ .

1. Show that  $\text{Ker}(f) \subset \text{Ker}(f^2)$ .

2. Show that  $\text{Im}(f^2) \subset \text{Im}(f)$ .

3. Show that  $\text{Im}(f) \cap \text{Ker}(f) = \{0_E\} \iff \text{Ker}(f) = \text{Ker}(f^2)$ .

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**Exercise 5 (2 points)**

We are working in  $\mathbb{R}_2[X]$ . For each of the following questions, you must justify your answer.

1. Let  $\mathcal{B}_1 = \{X^2 + X; X + 3\}$ . Is it a spanning family of  $\mathbb{R}_2[X]$ ?

2. Let  $\mathcal{B}_2 = \{2; X + 1; 2X^2; X^2 + 3\}$ . Is it a linearly independent family of  $\mathbb{R}_2[X]$ ?

3. Let  $\mathcal{B}_3 = \{1; X + 1; X^2 + 2X\}$ . Is it a basis of  $\mathbb{R}_2[X]$ ?

### Exercise 6 (3 points)

Let  $a$  and  $b$  be two real numbers,  $A = \begin{pmatrix} 1 & a & b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$  and  $J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ .

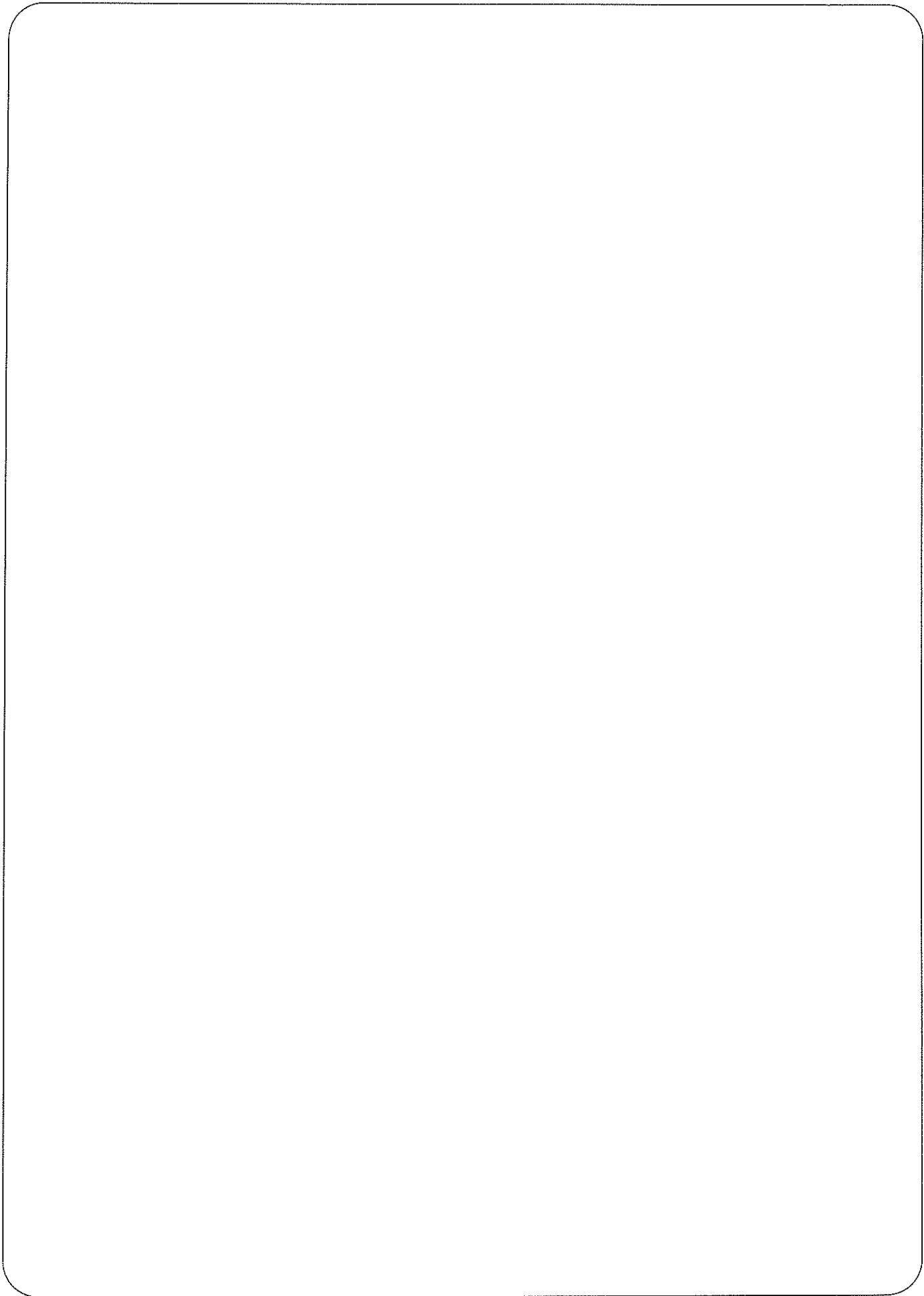
1. Calculate  $J^2$  and  $J^k$  for  $k \geq 3$ .

2. Express  $A$  as a function of  $I$ ,  $J$  and  $J^2$  where  $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ .

3. Deduce  $A^n$  for any  $n \in \mathbb{N}^*$ .

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### Exercise 7 (4 points)

$$\text{Let } f : \begin{cases} \mathbb{R}^4 & \longrightarrow \mathbb{R}^3 \\ (x, y, z, t) & \longmapsto (x + y, 2x + y + z, x + t) \end{cases}$$

1. Prove that  $f$  is a linear map.

2. Determine  $\text{Ker}(f)$  and give its dimension.

3. Deduce  $\text{Im}(f)$ .