EPITA / S_1 November 2017

NAME : .... FIRSTNAME :.. GROUP :.........

Physics Midterm 1

Calculators and extra-documents are not allowed.
Please answer only on exam sheets

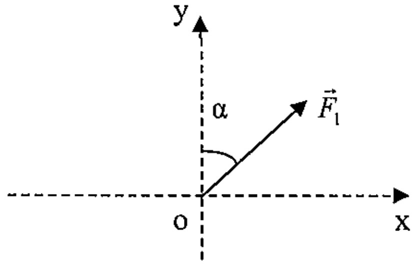
MCQ (4 points)

Circle the correct answer

1- The norm of the net force $\vec{R}$ of two non-vanishing forces $\vec{F}_1$ and $\vec{F}_2$, which are collinear and of opposite orientation, is

a) $R=0$ b) $R=\sqrt{F_1^2+F_2^2}$ c) $R=F_1+F_2$ d) $R=|F_1-F_2|$

2- The components of the vector force $\vec{F}_1$ sketched below are:



a) $\vec{F}_1=\begin{pmatrix}F_1\\0\end{pmatrix}$ b) $\vec{F}_1=\begin{pmatrix}F_1.\sin(\alpha)\\F_1.\cos(\alpha)\end{pmatrix}$ c) $\vec{F}_1=\begin{pmatrix}F_1.\cos(\alpha)\\F_1.\sin(\alpha)\end{pmatrix}$

3- The scalar product between two collinear vectors which have opposite orientation is

a) strictly positive b) vanishing c) strictly negative

4- The norm of the vector $\vec{V}_3=\vec{V}_1\wedge\vec{V}_2$, defined such that $(\vec{V}_1,\vec{V}_2)=\alpha$, is

a) $V_3=V_1.V_2.|\sin(\alpha)|$ b) $V_3=V_1.V_2.\cos(\alpha)$ c) $V_3=\sqrt{V_1^2+V_2^2+2V_1.V_2.\cos(\alpha)}$

5- The velocity vector reads in polar coordinates:

a) $\vec{V}=\dot{\rho}.\vec{u}_\rho+\dot{\theta}\,\vec{u}_\theta$ b) $\vec{V}=\dot{\rho}.\vec{u}_\rho+\rho\,\dot{\theta}\,\vec{u}_\theta$ c) $\vec{V}=\rho.\vec{u}_\rho+\dot{\theta}\,\vec{u}_\theta$

6- In Frenet's basis the elementary curvy coordinate ds reads:

a) $ds=R.\dot{\theta}$ b) $ds=dV.dt$ c) $ds=R.d\theta$

7- The expression of the curvy coordinate s(t) is given by

a) $s(t)=\int_0^t a_T.dt$ b) $s(t)=\int_0^t v.dt$ c) $s(t)=\int_0^t a_N.dt$

8- The trajectory equation of the motion whose time-dependent equations are $\begin{pmatrix}x(t)=A\sin(\omega.t)\\y(t)=B\cos(\omega.t)\end{pmatrix}$ (where A, B and ω are positive constants $(A\neq B)$) is:

a) $x^2+y^2=1$ b) $x^2+y^2=A^2+B^2$ c) $\dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$

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Exercise 1 (4 points)

The time-dependent equations of a motion are given in Cartesian coordinates:

$$\begin{cases} x(t) = 1 + R\sin(\omega.t) \\ y(t) = 2 + R\cos(\omega.t) \end{cases}$$ where ω and R are constants.

1-Express the components of the velocity vector $\vec{V}$ as a function of time. Write its norm.

2- Express the components of the acceleration vector $\vec{a}$ as a function of time. Write its norm.

3- Find the trajectory equation y = f(x). Describe its shape and its features.

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Exercise 2 (6 points)

The components of the vector position $\vec{OM}$ are written in Cartesian coordinates as:

$$\begin{cases} x(t) = ae^{\omega.t}\cos(\omega.t) \\ y(t) = ae^{\omega.t}\sin(\omega.t) \end{cases}$$ where a and ω are positive constants.

1- Write the position vector $\vec{OM}$ in polar coordinates in the basis $(\vec{u}_\rho, \vec{u}_\theta)$.

2- Express in polar coordinates the velocity vector $\vec{V}$. Write its norm. Assume that $\dot{\theta} = \omega$.

3- Express in polar coordinates the acceleration vector $\vec{a}$. Write its norm.

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4- Express the acceleration components a_T and a_N in Frenet's basis. Deduce the curvature radius Rc.

Exercise 3 Parts I and II are independent (6 points)

I-1) Prove that in Frenet's basis the velocity reads $\vec{V} = R(t)\dot{\theta}.\vec{u}_T$.

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I-2) Deduce the expression of the acceleration vector $\vec{a}$ in Frenet's basis.

II- Let us consider a pointlike mass M which is moving in a plan, with an acceleration given as a function of time in Frenet's basis by the following expression:

$$\vec{a} = \alpha.\vec{u}_T + \beta t^2.\vec{u}_N \qquad (\alpha \text{ and } \beta \text{ are positive constants})$$

1) Find the units of the constants α and β. Detail your answer.

2) Write the curvy coordinate s(t) between the moments $t_0 = 0$ and t. Given data: $v(t_0) = 0$ and $s(t_0) = 0$

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3) Prove that the curvature radius is given by: $R_c = \frac{\alpha^2}{\beta}$

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