

**EPITA**

**Mathematics**

**Midterm exam (S1)**

**November 2017**

**Name :**

**First Name :**

**Class :**

**MARK :**



# Midterm exam (S1)

Duration : three hours  
Documents and calculators not allowed

## Instructions :

- you have to reply directly on these sheets.
- *No sheet other than the stapled ones provided for answering will be corrected.*
- Answers written using lead pencils will not be corrected.
- Every student failing to respect these instructions will be awarded a 00/20 mark.

## Exercise 1 (2 points)

Let  $f$  and  $g$  be the functions defined by 
$$\begin{cases} f(x) = \sqrt{\ln^{10}(\sin(x)) + 1} \\ g(x) = \sin(\arctan(\sqrt{x})) \end{cases}$$

Calculate  $f'(x)$  and  $g'(x)$  (no need to refer to domains of definition).

N.B. : do not try to simplify the results.

## Exercise 2 (3 points)

Let  $z = 1 + \sqrt{3} + i(1 - \sqrt{3})$ .

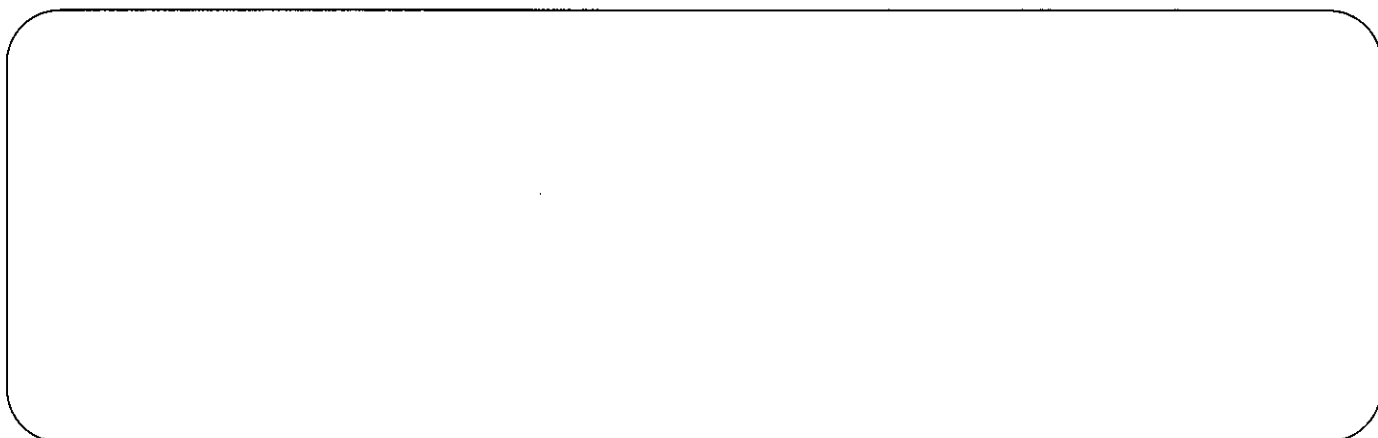
1. Determine  $z^2$  over exponential form.

2. Deduce from this the modulus and an argument of  $z$ .

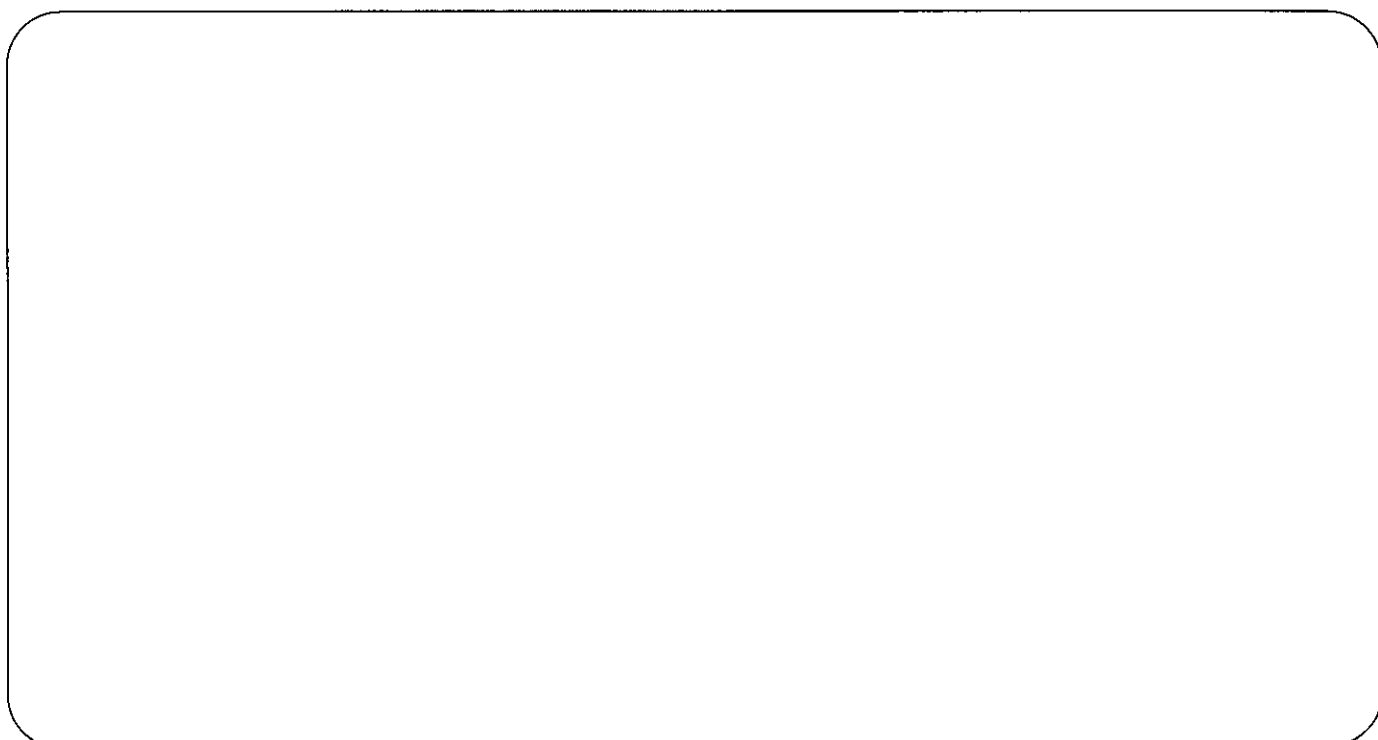


### Exercise 3 (6 points)

1. Determine, using neither an integration by parts nor a substitution,  $I = \int_0^1 \frac{\arctan(x)}{1+x^2} dx$ .



2. Using an integration by parts, determine  $J = \int_1^e \frac{\ln(x)}{x^2} dx$ .



3. Using the substitution  $u = \ln(t)$ , determine  $K = \int_1^e \frac{dt}{t(1 + \ln^2(t))}$ .

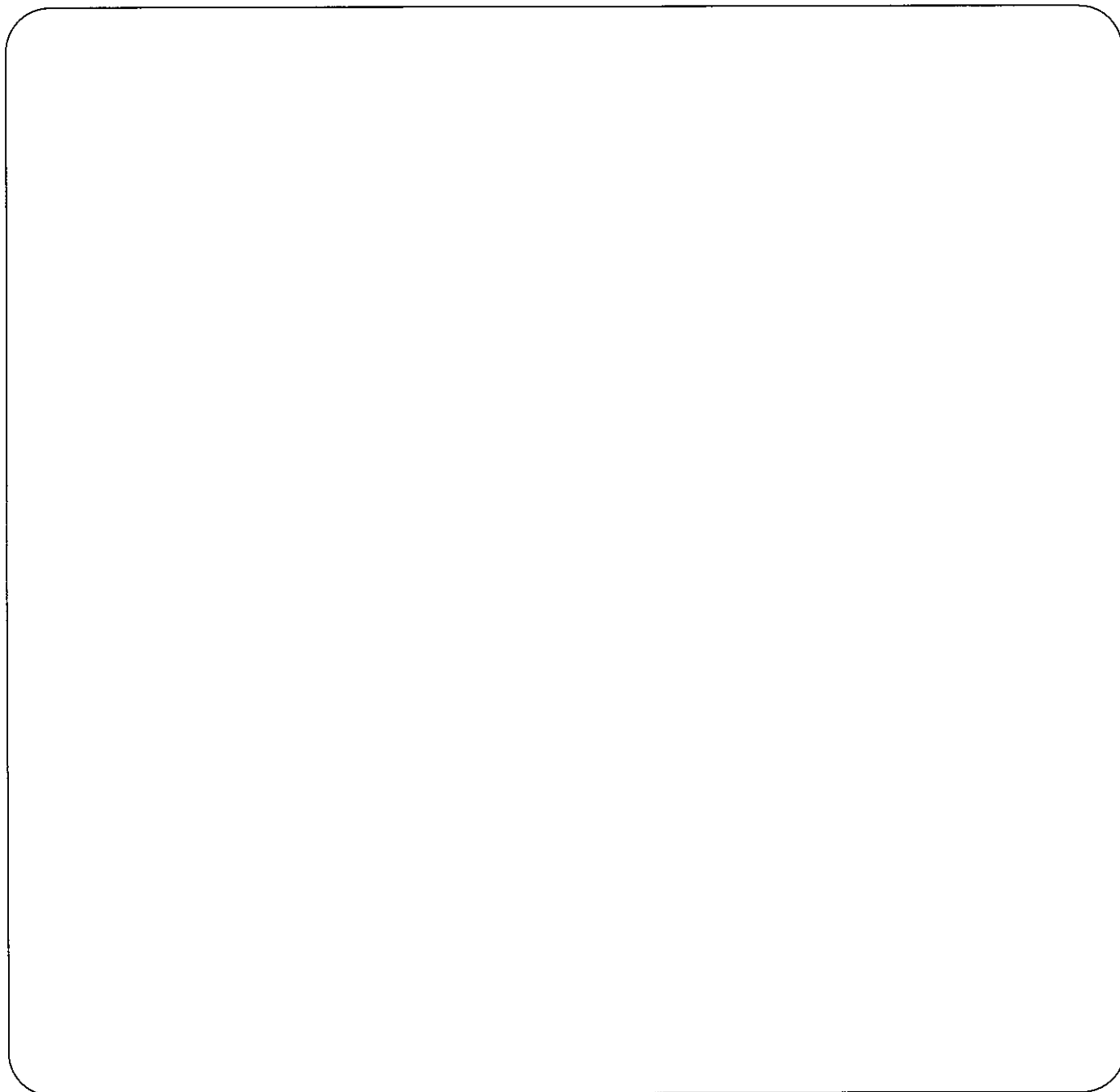
4. Using the substitution  $u = \sqrt{x}$ , determine  $L = \int_0^1 \frac{1-x}{1+\sqrt{x}} dx$ .

#### Exercise 4 (4 points)

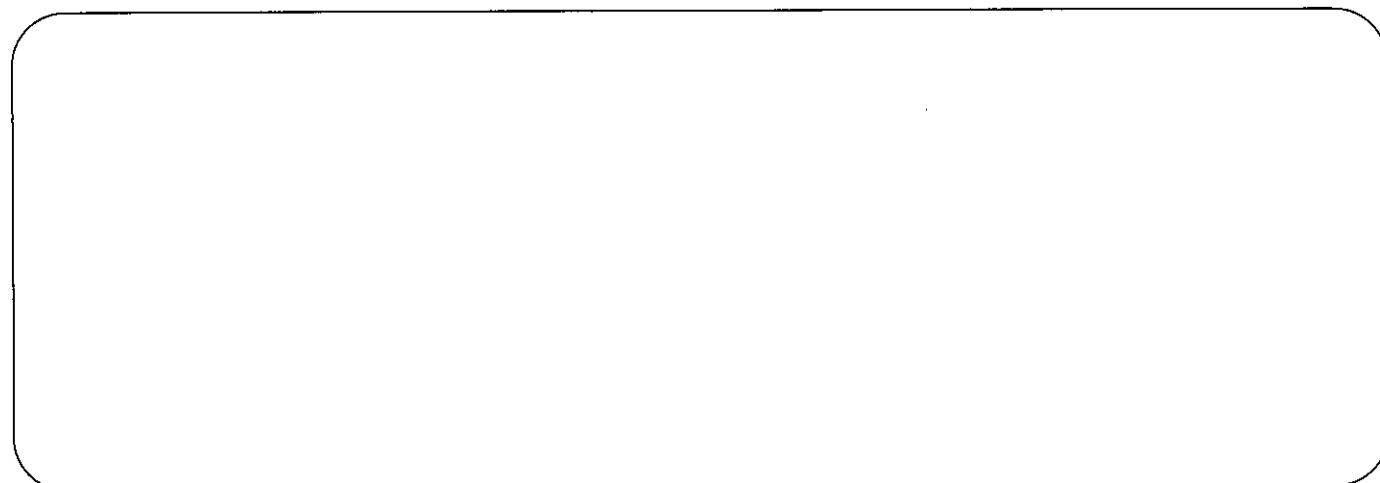
Let  $(E)$  be the following equation :  $z^2 - (5 + 3i)z + 2 + 9i = 0$ .

1. Show that  $\Delta = 8 - 6i$ .

2. Determine a square root of  $\Delta$ .



3. Deduce from this the solutions in  $\mathbb{C}$  of the equation  $(E)$ .



### Exercise 5 (4 points)

1. Determine the Taylor expansion around 0 at order 2 of  $e^x \ln(e + ex)$ .

2. Determine  $\lim_{x \rightarrow 0} (1 + \sin(x))^{1/x}$ .

3. Determine  $\lim_{x \rightarrow 0} \frac{e^x - \cos(x) - \sin(x)}{x^2}$ .

**Exercise 6 (2 points)**

Let  $f : [0, 1] \rightarrow \mathbb{R}$  be a continuous function such that  $f(0) = f(1)$ .

Show that there exists  $c \in \left[0, \frac{1}{2}\right]$  such that  $f(c) = f\left(c + \frac{1}{2}\right)$ .