EPITA

Mathematics

Midterm exam (S1)

November 2017

Name :

First Name :

Class :

MARK :

Midterm exam (S1)

Duration : three hours
Documents and calculators not allowed

Instructions :

- you have to reply directly on these sheets.
- ***No sheet other than the stapled ones provided for answering will be corrected.***
- Answers written using lead pencils will not be corrected.
- Every student failing to respect these instructions will be awarded a 00/20 mark.

Exercise 1 (2 points)

Let f and g be the functions defined by $\begin{cases} f(x) = \sqrt{\ln^{10}\left(\sin(x)\right) + 1} \\ \\ g(x) = \sin\left(\arctan(\sqrt{x})\right) \end{cases}$

Calculate $f'(x)$ and $g'(x)$ (no need to refer to domains of definition).

N.B. : do not try to simplify the results.

Exercise 2 (3 points)

Let $z = 1 + \sqrt{3} + i(1 - \sqrt{3})$.

1. Determine z^2 over exponential form.

2. Deduce from this the modulus and an argument of z.

Exercise 3 (6 points)

1. Determine, using neither an integration by parts nor a substitution, $I = \displaystyle\int_0^1 \frac{\arctan(x)}{1+x^2} \mathrm{d}x$.

2. Using an integration by parts, determine $J = \displaystyle\int_1^e \frac{\ln(x)}{x^2} \, \mathrm{d}x$.

3. Using the substitution $u = \ln(t)$, determine $K = \displaystyle\int_1^e \frac{\mathrm{d}t}{t(1+\ln^2(t))}$.

4. Using the substitution $u = \sqrt{x}$, determine $L = \displaystyle\int_0^1 \frac{1-x}{1+\sqrt{x}}\,\mathrm{d}x$.

Exercise 4 (4 points)

Let (E) be the following equation : $z^2 - (5+3i)z + 2 + 9i = 0$.

1. Show that $\Delta = 8 - 6i$.

2. Determine a square root of Δ.

3. Deduce from this the solutions in $\mathbb{C}$ of the equation (E).

Exercise 5 (4 points)

1. Determine the Taylor expansion around 0 at order 2 of $e^x \ln(e + ex)$.

2. Determine $\lim_{x \to 0} \left(1 + \sin(x)\right)^{1/x}$.

3. Determine $\lim_{x \to 0} \dfrac{e^x - \cos(x) - \sin(x)}{x^2}$.

Exercise 6 (2 points)

Let $f : [0,1] \longrightarrow \mathbb{R}$ be a continuous function such that $f(0) = f(1)$.
Show that there exists $c \in \left[\ 0,\ \frac{1}{2}\ \right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.