Key to Midterm Exam S1
Computer Architecture

Answer on the worksheet **Duration: 1 hr 30 min.**

Last name: ..... First name: ..... Group: .....

Exercise 1 (3 points)

Simplify the following expressions. Give each result in a power-of-two form. Write down the result only (do not show any calculation).

Expression	Result
$\frac{128^3 \cdot 4^7 \cdot 4^6}{(512^{-3} \cdot 64^3)^{-3}}$	$\mathbf{2^{20}}$
$\frac{(32^4 \cdot 1024^{-3}) \cdot (190+66)^{-5}}{(2^{-12} \cdot (2^8 - 2^7))^5 \cdot 128^{-4}}$	$\mathbf{2^{3}}$
$\frac{((4096 \cdot 8^{15})^3 \cdot 16384^{-5})^4}{(16^{-4} \cdot 2048)^{-7} \cdot 65536}$	$\mathbf{2^{353}}$

Exercise 2 (3 points)

1. How many bits do the following values contain? **Use a power-of-two notation**. Write down the result only (do not show any calculation).
   - 8 Kib = **2^{13} bits**
   - 256 Mib = **2^{28} bits**
   - 256 KiB = **2^{21} bits**

2. How many bytes do the following values contain? Use binary prefixes (Ki, Mi or Gi). **Choose the most appropriate prefix so that the integer numerical value will be as small as possible**. Write down the result only (do not show any calculation).
   - 128 Mib = **16 MiB**
   - 2^{41} bits = **256 GiB**
   - 2^{23} bytes = **8 MiB**

Exercise 3 (4 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not ¼ or 2^{-2}). Write down the result only (do not show any calculation).

Number to Convert	Source Form	Destination Form	Result
10101101.0011	Binary	Decimal	**173.1875**
D7.3	Hexadecimal	Decimal	**215.1875**
141.3	Decimal	Hexadecimal (2 digits after the point)	**8D.4C**
87.1875	Decimal	Binary	**101 0111.0011**
3071.16	Base 8	Hexadecimal	**639.38**
104.28	Decimal	Base 5 (2 digits after the point)	**404.12**
42	Base 9	Base 3	**1102**
1011101101.0110111	Binary	Hexadecimal	**2ED.6E**

Exercise 4 (2 points)

1. Work out the value of the base *b* so that the identity below is true. **Show all calculations.**

$124_b = 103_8$ **b > 4**

$b^2 + 2b + 4 = 8^2 + 3$
$b^2 + 2b + 4 = 67$
$b^2 + 2b - 63 = 0$

$\Delta = 4 + 4 \times 63 = 256$
~~b1~~ $= (-2 - 16) / 2 = -18 / 2 = -9$
b2 $= (-2 + 16) / 2 = 14 / 2 = 7$

b = 7

2. According to the identity below, determine the relation between the *a* and *b* bases and work out their smallest values. **Show all calculations.**

$208_a = 808_b$ **a > 8** and **b > 8**

$2a^2 + 8 = 8b^2+8$
$2a^2 = 8b^2$
$a^2 = 4b^2$
a = 2b

b_{min} = 9
a_{min} = 18

Exercise 5 (4 points)

Perform the operations below. **Show all calculations.**

Base 2

		1	0	0	0	1	1	0	1	0
	–		1	0	1	0	1	1	0	1
				1	**1**	**0**	**1**	**1**	**0**	**1**

Base 16

		3	C	2	8	
+		D	8	A	4	
	1	**1**	**4**	**C**	**C**	

Base 2

	1	0	0	1	1	0	1	0	1	1	1	0	
–		1	1	1	0				**1**	**0**	**1**	**1**	
			1	0	1	0	1						
		–		1	1	1	0						
					1	1	1	0					
				–	1	1	1	0					
								0					

Base 8

		7	6	5	4	
+		3	2	1	0	
	1	**3**	**0**	**6**	**4**	

Exercise 6 (4 points)

1. In terms of n, how many n-bit unsigned integers can be encoded?

2^n

2. In terms of n, how many n-bit signed integers can be encoded?

2^n

3. In terms of n, what is the largest n-bit unsigned integer that can be encoded?

2^n-1

4. In terms of n, what is the largest n-bit signed integer that can be encoded?

$2^{n-1}-1$

5. In terms of n, what is the smallest n-bit signed integer that can be encoded?

-2^{n-1}

Feel free to use the blank space below if you need to: