

Key to Midterm Exam S1

Computer Architecture

Answer on the worksheet

Duration: 1 hr. 30 min.

Last name: First name: Group:

Exercise 1 (3 points)

Simplify the following expressions. Give each result in a power-of-two form. Write down the result only (do not show any calculation).

Expression	Result
$\frac{64^3 \cdot 4^7 \cdot 16^9}{(4,096^{-5} \cdot 32^3)^7}$	2^{383}
$\frac{(64^8 \cdot 512^{-5}) \cdot (499+13)^{-9}}{(2^{-16} \cdot (2^{12} - 2^{11}))^4 \cdot 8,192^{-9}}$	2^{59}
$\frac{((16,384 \cdot 8^{13})^6 \cdot 65,536^{-4})^4}{(4^{-4} \cdot 256)^{-6} \cdot 32,768}$	2^{1001}

Exercise 2 (3 points)

1. How many bits do the following values contain? Use a power-of-two notation. Write down the result only (do not show any calculation).

• 256 Kib = 2^{18} bits

• 1 GiB = 2^{33} bits

• 512 MiB = 2^{32} bits

2. How many bytes do the following values contain? Use binary prefixes (Ki, Mi or Gi). Choose the most appropriate prefix so that the integer numerical value will be as small as possible. Write down the result only (do not show any calculation).

• 64 Mib = 8 MiB

• 2^{22} bits = 512 KiB

• 2^{37} bytes = 128 GiB

Exercise 3 (4 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not $\frac{1}{4}$ or 2^{-2}). Write down the result only (do not show any calculation).

Number to Convert	Source Form	Destination Form	Result
10011101.1001	Binary	Decimal	157.5625
1AD.9	Hexadecimal	Decimal	429.5625
515.3	Decimal	Hexadecimal (2 digits after the point)	203.4C
78.6875	Decimal	Binary	1001110.1011
427.316	Base 8	Hexadecimal	117.67
9.99	Decimal	Base 7 (3 digits after the point)	12.663
24	Base 9	Base 3	211
1010101111.10101	Binary	Hexadecimal	2AF.A8

Exercise 4 (2 points)

1. Work out the value of the base b so that the identity below is true. **Show all calculations.**

$$111_b = 1121_3 \quad b > 2$$

$$b^2 + b + 1 = 3^3 + 3^2 + 3 \times 2 + 1$$

$$b^2 + b + 1 = 43$$

$$b^2 + b - 42 = 0$$

$$\Delta = 1 - 4 \times (-42) = 169$$

$$b_1 = (-1 - 13) / 2 = -14 / 2 = -7$$

$$b_2 = (-1 + 13) / 2 = 12 / 2 = 6$$

$$\mathbf{b = 6}$$

2. According to the identity below, determine the relation between the a and b bases and work out their smallest values. **Show all calculations.**

$$200_a = 20402_b \quad a > 2 \text{ and } b > 4$$

$$2a^2 = 2b^4 + 4b^2 + 2$$

$$a^2 = b^4 + 2b^2 + 1$$

$$a^2 = (b^2 + 1)^2$$

$$a = b^2 + 1$$

$$b_{\min} = 5$$

$$a_{\min} = 26$$

Exercise 5 (4 points)

Perform the operations below. **Show all calculations.**

Base 2

		1	0	0	1	0	1	0	1	0
	–		1	0	1	1	1	0	1	1
				1	1	0	1	1	1	1

Base 16

		C	3	A	B	
+		5	E	A	9	
	1	2	2	5	4	

Base 2

	1	0	0	0	1	1	1	1	1	0	1	1	
−		1	0	1	1					1	1	0	1
			1	1	0	1							
		−	1	0	1	1							
					1	0	1	1					
				−	1	0	1	1					
									0				

Base 8

		5	6	3	3	
+		7	2	4	7	
	1	5	1	0	2	

Exercise 6 (4 points)

1. In terms of n , how many n -bit unsigned integers can be encoded?

$$2^n$$

2. In terms of n , how many n -bit signed integers can be encoded?

$$2^n$$

3. In terms of n , what is the largest n -bit unsigned integer that can be encoded?

$$2^n - 1$$

4. In terms of n , what is the largest n -bit signed integer that can be encoded?

$$2^{n-1} - 1$$

5. In terms of n , what is the smallest n -bit signed integer that can be encoded?

$$-2^{n-1}$$

Feel free to use the blank space below if you need to: