NAME : ..... FIRSTNAME :..... GROUP :.....

Physics Midterm n°1

Calculators and documents are not allowed.
Please answer only on exam sheets

Exercise 1 *Cycloidal motion* (7 points)

Part A

One considers Cartesian basis (Oxyz). One studies a wheel of radius R and center C which is rolling without gliding on plan (xOy) : it is admitted that the position of wheel center is linked to the angle θ describing the wheel rotation.
Coordinates of vector position can be expressed as :

$$\begin{cases} x(t) = A(\omega.t - \sin(\omega.t)) \\ y(t) = A(1 - \cos(\omega.t)) \end{cases} \quad (\theta = \omega.t)$$ *; where A and ω are positive constants.*

1- Write the Cartesian components of velocity and acceleration vectors.

2- Deduce norms of both vectors. Useful formula : $1 - \cos(\alpha) = 2.\sin^2(\alpha/2)$.

3-Draw the cycloid (y = f(x)) over a time interval of 2 periods (2 T). Remember that ω is linked to the period T by $\omega = 2\pi/T$.
(Consider the values : t = 0 ; t = T/4 ; t = T/2 ; t = 3T/4 ; t = T).

Part B

One considers spiral motion described by the following equations :

$\begin{cases} \rho(t) = \rho_0 e^{\omega.t} \\ \theta(t) = \omega.t \end{cases}$; where ρ_0 and ω are positive constants.

1- Express the velocity vector of this motion in polar coordinates. One reminds you that :

$$\vec{V} = \dot{\rho}\,\vec{u}_\rho + \rho\,\dot{\theta}\,\vec{u}_\theta$$

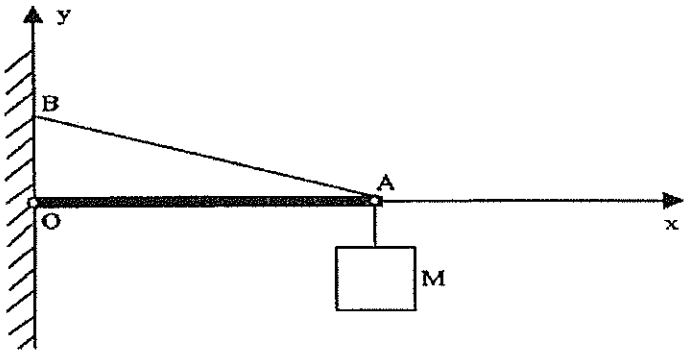
2- Write the norm of velocity vector.

3-a) Remembering that in Frenet's basis $\vec{V} = V.\vec{u}_T = R(t)\dot{\theta}\vec{u}_T$, express the radius R(t) of that trajectory.

b) Deduce from it the components of acceleration vector $\vec{a}(a_T, a_N)$ in Frenet's basis $(\vec{u}_T, \vec{u}_N)$.

Exercise 2 System at equilibrium (6 points)

A homogeneous horizontal beam OA of length L and mass m = 40 kg is fixed to a wall at its tip O. A cable AB, whose mass is neglected and length is fixed, connects the wall and the tip A of the beam. A mass M = 150 kg is hung at point A. Given data : BAO = 30° and g = 10m.s^{-2}.



1- Write explicitly which exterior forces are acting on the beam. Sketch them.

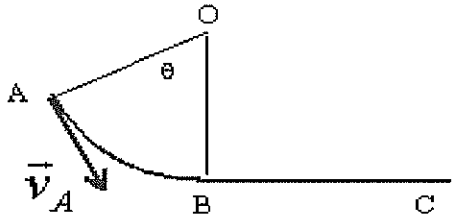
2- a) Write the rotation equilibrium condition with respect to point O and deduce then the norm of cable stress.

b) Use translation equilibrium condition to write the components (R_x, R_y) of $\vec{R}_{wall}$.

c) Compute the norm of the reaction $\vec{R}_{wall}$.

Exercise 3 (7 points)

A pointlike solid of mass m is moving on the path sketched below. Part AB is circular with radius R, angle θ and center O while BC is horizontal. The solid is thrown from point A with a velocity V_A tangential to the circle.



1-a) Write all exterior forces acting on solid between A and B by assuming that frictions over part AB can be modelized by a constant force f. Sketch them.

b) Use kinetic energy theorem between A and B in order to express the friction force f as function of R, g, V_A, V_B, m and θ. Compute numerically with m = 0,1kg, g =10ms^{-2}, R = 1,5m, V_A = 2ms^{-1}, V_B = 3ms^{-1} ; θ = 60° ≈ 1 rad.

2- a) Frictions over path BC can be modelized by a force f = 0,1 N. Compute the speed at point C with BC = 2m.

b) Compute the norm of the total reaction $\vec{R}$ which is acting on solid over path BC.