Final exam 1

Duration : three hours
Documents and calculators not allowed

Name : **First name :** **Class :**

Instructions :

– ***no sheets other than the stapled ones provided for answers will be corrected.***

– answers written using lead pencils will not be corrected.

Exercise 1 (2 points)

Write the negation of the following sentences :

1. « The square root of an even natural number is even ».
2. « For any triangle in the plane, the sum of the angles is equal to 180° in Euclidean geometry ».
3. « Some students will not leave for their semester abroad in S4 ».
4. « Some students will leave for their semester abroad in S4 ».

Exercise 2 (2 points)

Prove by induction that for any $n \geqslant 4$, $n! > 2^n$.

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Exercise 3 (2 points)

Let f be a function from $\mathbb{R}$ to $\mathbb{R}$. Write in mathematical language (using quantifiers) the following sentences :

1. « the function f has at least one root ».
2. « f is not the zero function ».
3. « f is the zero function ».
4. « f admits a minimum on $\mathbb{R}$ ».

Exercise 4 (2 points)

Let E be a set, $f : E \longrightarrow E$ and $g : E \longrightarrow E$.

1. We suppose that both f and g are injective. Show that $f \circ g$ is injective.
2. We suppose that both f and g are surjective. Show that $f \circ g$ is surjective.

3. Show that $g \circ f$ injective $\Longrightarrow$ f injective.

4. Show that $g \circ f$ surjective $\Longrightarrow$ g surjective.

Exercise 5 (3 points)

1. Using Euclid's algorithm, determine a particular solution of the equation $732x + 124y = 4$.

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2. Using imperatively Gauss's theorem, determine the set of all the ordered pairs $(x, y) \in \mathbb{Z}^2$ such that $732x + 124y = 4$.

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Exercise 6 (3 points)

Let $(a, b) \in \mathbb{N}^2$. Show that : $(a+b)$ and ab are coprime $\iff$ a and b are coprime.

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Exercise 7 (2 points)

What is the remainder of the Euclidean division of 12^{1527} by 5?

Exercise 8 (2 points)

Determine the order of multiplicity of the root 1 of the polynomial $P(X) = X^4 - 2X^3 + 2X - 1$.

Exercise 9 (3 points)

Let $n \in \mathbb{N}^*$.

1. Show that $X^2 + 2X$ divides $(X+1)^{2n} - 1$.

2. Show that X^2 divides $(X+1)^n - nX - 1$.

3. Show that $(X-1)^2$ divides $nX^{n+1} - (n+1)X^n + 1$.