## Key to Final Exam S1 Computer Architecture

Duration: 1 hr. 30 min.

Last name: $\qquad$ First name: $\qquad$ Group: $\qquad$

## Write answers only on the worksheet.

 Do not show any calculation unless you are explicitly asked.
## Do not use a pencil or red ink.

## Exercise 1 (2 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not $1 / 4$ or $2^{-2}$ ).

| Number to Convert | Source Form | Destination Form | Result |
| :---: | :---: | :---: | :---: |
| 101111011.01001 | Binary | Decimal | $\mathbf{3 7 9 . 2 8 1 2 5}$ |
| 2CD.48 | Hexadecimal | Decimal | $\mathbf{7 1 7 . 2 8 1 2 5}$ |
| 750 | Decimal | Base 6 | $\mathbf{3 2 5 0}$ |
| BB8.68 | Hexadecimal | Base 8 | $\mathbf{5 6 7 0 . 3 2}$ |

## Exercise 2 (5 points)

Perform the following 8 -bit binary operations (the two operands and the result are 8 bits wide). Then, convert the result into unsigned and signed decimal values. If an overflow occurs, write down 'ERROR' instead of the decimal value.

| Operation | Binary Result | Decimal Value |  |
| :---: | :---: | :---: | :---: |
|  |  | Signed |  |
| $10000111+10101101$ | 00110100 | ERROR | ERROR |
| $01010010-10001101$ | 11000101 | ERROR | ERROR |
| $00110111+10111000$ | 11101111 | $\mathbf{2 3 9}$ | $\mathbf{- 1 7}$ |
| $10000001-10000010$ | 11111111 | ERROR | $\mathbf{- 1}$ |
| $11000111-01101100$ | 01011011 | $\mathbf{9 1}$ | ERROR |

## Exercise 3 (5 points)

Amongst the great variety of binary encoding techniques, there is the 2421 code. In this code, the weights of the binary digits are $2,4,2,1$, instead of $8,4,2,1$. Therefore, several binary patterns are possible for some decimal numbers. For instance, the encoded value of $5_{10}$ can be either 0101 or 1011 . Furthermore, the encoded value of $9_{10}$ is made up of four ones: 1111. It means that, with four bits, no value greater than $9_{10}$ can be encoded in 2421 code (unlike the 8421 natural binary form, where values from $0_{10}$ to $15_{10}$ can be encoded).

## The Aiken code is a kind of 2421 code:

- The encoded values from 0 to 4 in Aiken code are identical to the encoded values from 0 to 4 in BCD code.
- The encoded values from 5 to 9 in Aiken code are identical to the encoded values from 11 to 15 in natural binary code.

We want to design a circuit that converts a 4-bit natural binary code (DCBA) into its 4-bit Aiken code (D'C’B'A'). Complete the following truth table and the Karnaugh maps below (draw also the circles). Then, give the most simplified expression for each output. When a solution is obvious, you do not have to complete its associated Karnaugh map. As a reminder, an obvious solution does not have any logical operations apart from the complement (for instance: $\mathrm{A}^{\prime}=1, \mathrm{~A}^{\prime}=\overline{\mathrm{A}}$ ).

| $\mathbf{D}$ | $\mathbf{C}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{D}$ | $\mathbf{C}$ | $\mathbf{B}$ | $\mathbf{A}^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| 0 | 0 | 1 | 0 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| 0 | 0 | 1 | 1 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| 0 | 1 | 1 | 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| 1 | 0 | 0 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

DC

| D' | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{0 1}$ | 0 | 1 | 1 | 1 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 1 | 1 | $\Phi$ | $\Phi$ |

$$
\mathbf{D}^{\prime}=\mathbf{D}+\mathbf{C} . \mathbf{A}+\mathbf{C} . \mathbf{B}
$$

DC

| $\mathbf{C}$ | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{0 1}$ | 1 | 0 | 1 | 1 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 1 | 1 | $\Phi$ | $\Phi$ |

$$
\mathbf{C}^{\prime}=\mathbf{D}+\mathbf{C} \cdot \overline{\mathbf{A}}+\mathbf{C} \cdot \mathbf{B}
$$

BA

DC | $\mathbf{B}$ | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 1 | 1 |
| $\mathbf{0 1}$ | 0 | 1 | 0 | 0 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 1 | 1 | $\Phi$ | $\Phi$ |

$\mathbf{B}^{\prime}=\mathbf{D}+\overline{\mathbf{C}} \cdot \mathbf{B}+\mathbf{C} \cdot \overline{\mathbf{B}} \cdot \mathbf{A}$

| BA |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ $\mathbf{0 0}$ $\mathbf{0 1}$ $\mathbf{1 1}$ $\mathbf{1 0}$ <br> $\mathbf{0 0}$     <br> $\mathbf{0 1}$     <br> $\mathbf{1 1}$     <br> $\mathbf{1 0}$     |  |  |  |  |  |  |

$\mathbf{A}^{\mathbf{\prime}}=\mathbf{A}$

## Exercise 4 (5 points)

For the whole exercise, write down the result only (do not show any calculation).
Let us consider the two following expressions:
$\mathrm{S} 1=\mathrm{A} \cdot \mathrm{B} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{B} \cdot \overline{\mathrm{C}}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \mathrm{C}$
$\mathrm{S} 2=\overline{\overline{\mathrm{A}} \cdot \mathrm{B}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}}$

1. Give the most simplified expressions of $S 1$ and $S 2$. The result must be given as a sum of products (without parentheses). Do not simplify by using the EXCLUSIVE-OR operator.
```
S1 = A.C + A.\overline{B}+\overline{\textrm{A}}\cdot\textrm{B}\cdot\overline{C}
```

$\mathrm{S} 2=\mathrm{A}+\overline{\mathrm{B}} . \mathrm{C}$
2. Simplify S1 by using the EXCLUSIVE-OR operator.
$\mathrm{S} 1=\mathrm{A} \oplus(\mathrm{B} . \overline{\mathrm{C}})$
3. Write down the maxterm canonical form of S1.
$S 1=(A+B+C) \cdot(A+B+\bar{C}) \cdot(\bar{A}+\bar{B}+C) \cdot(A+\bar{B}+\bar{C})$
4. Write down the minterm canonical form of S2.
$\mathrm{S} 2=\mathrm{A} \cdot \mathrm{B} \cdot \mathrm{C}+\mathrm{A} \cdot \mathrm{B} \cdot \overline{\mathrm{C}}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{B}} \cdot \mathrm{C}$

## Exercise 5 (3 points)

Perform the operations below. Show all calculations.

| Base 2 |  |  |  |  |  |  |  |  |  |  |  |  |  | Base 16 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 0 | 0 | 1 |  | 1 | 1 |  | 0 |  | 0 |  | 0 |  |  |  | 4 | 8 | A | 6 |  |
| - |  | 1 | 0 | 1 |  | 0 | 0 |  | 0 |  | 1 |  | 1 |  | + |  | E | 5 | C | 7 |  |
|  |  | 1 | 0 | 0 |  | 1 | 0 |  | 1 |  | 0 |  | 1 |  |  | 1 | 2 | E | 6 | D |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Base 2: tw | d | at |  | poin |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1 | 1 | 1 | 0 | 1 |  | 0 | 1 |  | 0 |  | 1 | 0 |  | 0 | 0 |  |  |  |  |  |
| - | 1 | 0 | 0 | 0 |  |  |  |  |  |  |  | 1 | 1 |  | 1 | 0 | 1 | - | 0 | 1 |  |
|  |  | 1 | 1 | 0 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | - | 1 | 0 | 0 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 | 0 | 1 |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | - | 1 | 0 | 0 |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 1 |  | 0 | 1 |  | 0 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | - | 1 |  | 0 | 0 |  | 0 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 1 |  | 0 |  | 0 | 0 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | - | 1 |  | 0 |  | 0 | 0 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Feel free to use the blank space below if you need to:

