Key to Final Exam S1
Computer Architecture

Duration: 1 hr. 30 min.

Last name: .... First name: .... Group: ....

Write answers only on the worksheet.
Do not show any calculation unless you are explicitly asked.
Do not use red ink.

Exercise 1 (2 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not ¼ or 2^{-2}).

Number to Convert	Source Form	Destination Form	Result
11011001.0011	Binary	Decimal	**217.1875**
BC.3	Hexadecimal	Decimal	**188.1875**
18	Decimal	Base 5	**33**
1111000111.11011	Binary	Hexadecimal	**3C7.D8**

Exercise 2 (5 points)

Perform the following 8-bit binary operations (the two operands and the result are 8 bits wide). Then, convert the result into unsigned and signed decimal values. If an overflow occurs, write down 'ERROR' instead of the decimal value.

Operation	Binary Result	Decimal Value	
		Unsigned	Signed
01100010 - 10011010	11001000	**ERROR**	**ERROR**
11111111 + 11111111	11111110	**ERROR**	**-2**
01111111 + 00000001	10000000	**128**	**ERROR**
10010010 - 10000101	00001101	**13**	**13**
11111111 - 11111111	00000000	**0**	**0**

Exercise 3 (6 points)

Let us consider *N*, a BCD number: *N* = *DCBA*. We want to design a circuit that multiplies *N* by 4. The result should be encoded in a BCD form and so be made up of 2 digits: *H'G'F'E'* for the tens column and *D'C'B'A'* for the units column (the MSBs are the leftmost bits). Complete the truth table and the Karnaugh maps below (**draw also the circles**). Then, give the most simplified expression for each output (**do not simplify by using the EXCLUSIVE-OR operator**). Three solutions are obvious. When a solution is obvious, you do not have to complete its associated Karnaugh map. An obvious solution does not have any logical operations apart from the complement (for instance: A' = 1, A' = $\overline{\text{A}}$).

					Tens Column				Units Column			
N	**D**	**C**	**B**	**A**	**H'**	**G'**	**F'**	**E'**	**D'**	**C'**	**B'**	**A'**
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	1	0	0
2	0	0	1	0	0	0	0	0	1	0	0	0
3	0	0	1	1	0	0	0	1	0	0	1	0
4	0	1	0	0	0	0	0	1	0	1	1	0
5	0	1	0	1	0	0	1	0	0	0	0	0
6	0	1	1	0	0	0	1	0	0	1	0	0
7	0	1	1	1	0	0	1	0	1	0	0	0
8	1	0	0	0	0	0	1	1	0	0	1	0
9	1	0	0	1	0	0	1	1	0	1	1	0

Obvious Solutions		
H'	**G'**	**A'**
0	0	0

BA

	F'	**00**	**01**	**11**	**10**
	00	0	0	0	0
DC	**01**	0	1	1	1
	11	Φ	Φ	Φ	Φ
	10	1	1	Φ	Φ

F' = D + C.A + C.B

BA

	E'/B'	**00**	**01**	**11**	**10**
	00	0	0	1	0
DC	**01**	1	0	0	0
	11	Φ	Φ	Φ	Φ
	10	1	1	Φ	Φ

E' = B' = D + C.$\overline{\text{B}}$.$\overline{\text{A}}$ + $\overline{\text{C}}$.B.A

BA

	D'	**00**	**01**	**11**	**10**
	00	0	0	0	1
DC	**01**	0	0	1	0
	11	Φ	Φ	Φ	Φ
	10	0	0	Φ	Φ

D' = C.B.A + $\overline{\text{C}}$.B.$\overline{\text{A}}$

BA

	C'	**00**	**01**	**11**	**10**
	00	0	1	0	0
DC	**01**	1	0	0	1
	11	Φ	Φ	Φ	Φ
	10	0	1	Φ	Φ

C' = C.$\overline{\text{A}}$ + $\overline{\text{C}}$.$\overline{\text{B}}$.A

Finally, simplify one of the outputs by using the EXCLUSIVE-OR operator:

D' = C.B.A + $\overline{\text{C}}$.B.$\overline{\text{A}}$ = B.(C.A + $\overline{\text{C}}$.$\overline{\text{A}}$) = **B . $\overline{\text{C} \oplus \text{A}}$**

Exercise 4 (4 points)

Let us consider the two following expressions:

$S1 = Y.(X + \overline{Z}) + (\overline{X} + Z).(X + Z)$

$S2 = (\overline{Y} + Z).(\overline{X} + \overline{Z}).(\overline{X} + Y + Z)$

1. Give the most simplified expressions of *S1* and *S2*. **The result must be given as a sum of products.**

$S1 = Y + Z$

$S2 = \overline{X}.Z + X.\overline{Y}$

2. Write down the minterm canonical form of *S1* (from the most simplified expression).

$S1 = Y.Z + Y.\overline{Z} + \overline{Y}.Z$

3. Write down the maxterm canonical form of *S2*.

$S2 = (X + \overline{Y} + Z).(\overline{X} + \overline{Y} + Z).(\overline{X} + Y + \overline{Z}).(\overline{X} + \overline{Y} + \overline{Z}).(\overline{X} + Y + Z)$

Exercise 5 (3 points)

Let us consider the following number: 2^{22}

1. Assuming that it is unsigned, determine the minimum number of bits required to encode it.

23 bits are required.

2. Assuming that it is signed, determine the minimum number of bits required to encode it.

24 bits are required.

Let us consider the following number: -2^{22}

3. Assuming that it is signed, determine the minimum number of bits required to encode it.

23 bits are required.

Feel free to use the blank space below if you need to: